1. General Equation of an Ellipse

2. You should be familiar with the General Equation of a Circle and how to shift and stretch graphs, both vertically and horizontally.

   In this lesson, we will find the equation of an ellipse, given the graph.

3. (a) Recall that the equation of a circle centered at the origin is $x^2 + y^2 = r^2$
   (b) We can divide both sides by r^2 to get the right hand side equal to 1
   (c) and if we wish, rewrite the squares outside the parentheses. We do this to compare transformations of the circle to the transformations of waves.

4. (a) Recall that a multiplier inside the parentheses, with the x, makes the wave thinner.
   (b) If instead, we divide, the wave will become wider.
   (c) The same is true for the circle. We can think of a circle with radius r as a circle of radius 1 that has been stretched by a factor of r both horizontally and vertically.

5. (a) An ellipse is a circle that has been stretched unequally. We can stretch by a factor of a in the x-direction and a factor of b in the y-direction.
   (b) We can also write the equation of an ellipse without the parentheses. The last equation is the standard form of an ellipse, centered at the origin.

6. (a) Here is an example. This ellipse is a standard circle, centered at the origin, that has been stretched 3 times wider and 5 times taller. You may wish to pause the video to write out the equation.
   (b) The horizontal stretch is by a factor of 3, so the number under the x^2 is 3^2, which is 9, and the number under the y^2 is 5^2, since the ellipse is stretched by a factor of 5 vertically.

7. The points on the ellipse that are farthest apart are called the major vertices and the line segment connecting them is called the major axis. The points on the ellipse that are closest together are called the minor vertices and the line segment connecting them is called the minor axis. The intersection of the major axis and minor axis is called the center.

8. (a) Now let's graph the ellipse

$$\frac{x^2}{16} + \frac{y^2}{36} = 1$$

   (b) This equation is in standard form. The right side of the equation is 1. $16 = 4^2$, so the circle has been stretched a factor of 4 in the x-direction, putting the vertices at $(4, 0)$ and $(-4, 0)$.
   (c) $36 = 6^2$ so the circle has been stretched a factor of 6 in the y-direction, putting vertices at $(0, 6)$ and $(0, -6)$.
   (d) We can now sketch the ellipse.
   (e) .

9. As we did with the circle, we can shift the center to the point (h, k). We will have vertices to the left and right of the center by a distance of a, and vertices above and below center by a distance of b.

10. (a) For example, let's graph $9(x-3)^2+16(y+2)^2=144$.
    (b) We need the right side equal to 1, so first we divide by 144 to get the ellipse in standard form.
    (c) The center is at $(3,-2)$. There will be vertices 4 to the left and right since the x^2 term is divided by 4^2
    (d) and there will be vertices 3 above and below center since the y^2 term is divided by 3^2.
    (e) We can then sketch the ellipse
    (f) .
11. (a) Let's find the equation for this ellipse. The center is at $(-2,-1)$, so the numerators will be $(x+2)^2$ and $(y+1)^2$ respectively.
    (b) From the center, we move left and right 4, so $a^2=4^2=16$.
    (c) From the center, we move up and down 6, so $b^2=6^2=36$
    (d) This is the standard equation of the ellipse.
12. To recap: When written in standard form, the center of an ellipse is at (h,k). The vertices in the x-direction are a distance of a from the center, and the vertices in the y-direction are a distance of b from the center.