1. General Equation of a Parabola

2. You should be familiar with the graph of the quadratic function $y = x^2$, as well as transformations of graphs, specifically how to shift, stretch and flip a graph.

   In this lesson, we will write the equation of a parabola, given its graph.

3. The standard parabola is $y = x^2$. The y-axis is the axis of symmetry, the parabola looks the same to the left of the y-axis as it does to the right of the y-axis. The point on the axis of symmetry which divides the parabola into two equal branches is called the vertex, which in this case, is at the origin. From the vertex, there are points that go left and right 1 and up 1, and also points that go left and right 2, up 4. We will use these points $(1, 1)$ and $(2, 4)$ to help us with other graphs.

4. If we change the roles of x and y, we get a parabola which is pointed to the right, symmetric across the x-axis. From the vertex, we can go up or down 1 and right 1. We can go up or down 2 and right 4. Here, we are thinking of y as the input, and then squaring to get the x value.

5. (a) We can now perform the standard transformations of graphs. First, we can reflect the graph across the x-axis by placing a negative in front of the x^2.
   (b) The red parabola has negative values for the y-coordinates.

6. (a) We can stretch the parabola in two different ways. The first is similar to amplitude for sine waves. Multiplying a number outside the parentheses, after the function of squaring is applied, will make the graph taller.
   (b) $y = 4x^2$ makes the y-coordinates four times as large.

7. (a) Here is the same parabola, this time we will multiply inside the parentheses. This transformation is similar to the frequency and wavelength concept for sine waves.
   (b) Multiplying by 2 inside the parentheses makes the graph two times thinner. The point at $(2, 4)$ moved closer to the y-axis to $(1, 4)$. You may wish to rewind the video at this point to view these last two examples again. You will notice that the equations are the same equation, written in a different way. The graphs are the same graphs. We can think of the stretching as happening either in the x-direction or in the y-direction. The end result can be accomplished in two different ways.

8. (a) From now on, we will always take the amplitude point of view. We will put the multiplier outside the parentheses, and think of the stretching happening parallel to the axis of symmetry.
   (b) A minus sign in front will reflect the graph, the A will stretch the graph vertically. The h will shift the graph horizontally and the k will shift the graph vertically, so the vertex will be at (h, k).

9. (a) Let's graph this parabola. It is a y equals x-squared parabola, so it is oriented vertically. The negative sign will make it point downward. It is not in standard form, we should subtract 7 from both sides.
   (b) We now see the vertex will be at $(-3, 7)$

(c) The parabola points downward and is stretched by a factor of 2. Instead of going right 1 and down 1, we will go right one and down 2 to the point $(-2, 5)$

(d) Similarly, one left and two down puts us at $(-4, 5)$. We can also find points that went left or right 2, and down twice the usual 4, or down 8 to -1, placing points at $(-1, -1)$ and $(-5, -1)$.

(e) We can then draw the parabola.

(f) .

10. (a) Now let's write the equation, given the graph. The vertex is at $(1, -4)$, so to the standard equation $y = x^2$, we will subtract 1 from x and add 4 to y.

    (b) The parabola opens upward, so A is positive. When we move left or right 1, we expect to go 1, but in this case, we went up 2, so the stretch factor is 2.

    (c) The equation is $y + 4 = 2(x - 1)^2$.

11. For the parabola in the horizontal orientation, the standard form is similar. The h shifts the parabola in the x-direction and the k shifts the graph in the y-direction. A minus sign will reflect the graph horizontally, so that it now points to the left. The A will stretch the graph horizontally, making the x-distances larger by a factor of A.

12. (a) Here is an example of a horizontal parabola. The standard form is $x = y^2$. It opens to the right, so it is positive y^2.

    (b) The vertex is at $(-4, 2)$, so $h = -4$ and $k = 2$.

    (c) We can determine the stretch factor as we did in example one, but here is another approach. By taking a point on the parabola, like $(2, 4)$

    (d) and plugging in the values of 2 for x and 4 for y, we now have just one variable in the equation. We can solve for A to obtain the value $3/2$.

    (e) We can also find the stretch factor as we did in Example 1: From the vertex with y-coordinate 2, we go up to 4, a distance of 2. The standard $x = y^2$ parabola moves 2 from the vertex and right 4. We expect to go right 4 to the point $(0, 4)$. Instead, we went further, to the point $(2, 4)$, a distance of 6. The standard is 4, we went 6, so the stretch factor is $6/4$ or $3/2$

    (f) The equation of this parabola is

$$x + 4 = \frac{3}{2}(y - 2)^2$$

13. To recap: The $y = x^2$ parabola points upward, the $x = y^2$ parabola points to the right. A negative sign will flip the graph, so that $y = -x^2$ points downward and $x = -y^2$ points to the left. Once you have determined the general form of the parabola, you can shift it so that the vertex is at (h, k). The factor of A will stretch the graph.