1. General Equation of a Circle

2. You should be familiar with the Pythagorean Theorem and transformations of graphs, specifically shifting graphs left, right, up and down.

   In this lesson, we develop the equation of a circle from the geometric definition.

3. The geometric definition of a circle is the set of all points located a fixed distance from some fixed point. The distance is called the radius and the fixed point is called the center.

4. (a) How do we find the equation of a circle? If the circle is centered at the origin, we need to find a relation involving the arbitrary point (x, y). For this example, the radius is 5.

   (b) The x-coordinate measures the x-distance and the y-coordinate measures the y-distance. We have a right triangle, so the equation is merely the Pythagorean Theorem.

   (c) In this example, $x^2 + y^2 = 5^2$, which is 25.

5. In general, the equation of a circle centered at the origin is $x^2 + y^2 = r^2$

6. (a) What happens if we move the center of the circle away from the origin? In this example, the radius is the distance from the center $(3, 2)$ to an arbitrary point (x, y). You find the horizontal distance by subtracting x-coordinates and the vertical distance by subtracting y-coordinates.

   (b) The horizontal distance is $x - 3$ and the vertical distance is $y - 2$.

   (c) In this case, the Pythagorean Theorem gives us $(x - 3)^2 + (y - 2)^2 = 9$

7. In general, we have $(x - h)^2 + (y - k)^2 = r^2$

8. To recap: If you are given the center of a circle at (h, k) and radius r, the equation of the circle is the Pythagoream Theorem $(x - h)^2 + (y - k)^2 = r^2$.

9. Here are two sample problems to test your understanding.