

1. Double and Half Angle Formulas

2. You should be familiar with the angle sum formulas and should be able to derive the double angle formulas from the angle sum formulas. You should be familiar with inverse trig functions and should be able to simplify fractions containing square roots, including the technique of rationalizing the denominator.

In this lesson, you will use the formulas to find specific values.

3. (a) We can derive the double angle formulas from the angle sum formulas by letting both angles be the same letter. You may wish to try to derive the formulas in the left column now.

(b) Here are the formulas. Note that for $\sin\left(\frac{A}{2}\right)$ and $\cos\left(\frac{A}{2}\right)$, you will need to decide whether the answer should be positive or negative. You will make this decision based on information about the angle. For example, if the angle A is between 270° and 360° , then half of angle A will be between 135° and 180° , and will therefore be a second quadrant angle which will have a positive sine value, but a negative cosine value.

4. (a) To solve problems using these formulas, we use the same procedure as we did with the angle sum formulas.

(b) First, we are given information about $\tan\theta$, so we can draw the triangle and label the legs. Note that we are given that θ is in the third quadrant, so I drew the hypotenuse going down and to the left.

(c) Since θ is in the third quadrant, the sine and cosine will be negative, so perhaps it is better to label the legs with negative numbers. We can then use the Pythagorean Theorem to find the length of the hypotenuse.

(d) We can then read the values of sine, cosine and tangent from the triangle.

(e) and then plug them into the appropriate formulas. First the formula for $\sin 2\theta$.

(f) Next, the formula for $\tan 2\theta$.

5. and finally the formula for $\tan\frac{\theta}{2}$.

6. .

7. Here we have worked out the simplification for the previous example.

8. (a) We may also be given the values of trig functions via inverse trig functions, as we were for the angle sum formulas.

(b) We begin by naming the angle

(c) Finding the appropriate formula.

(d) Labelling a triangle. Note that the inverse tangent function picks an angle from the right half of the unit circle and is therefore either a first quadrant or fourth quadrant angle. In this case it is a first quadrant angle since the tangent value is positive.

(e) The Pythagorean Theorem finds the length of the hypotenuse.

(f) We then read the necessary values from the triangle.

(g) and plug them into the formula.

9. To recap: In most cases, you will be given the value of a single trig function and be asked to find the value of a trig function for the double or half angle. To do so, draw and label a triangle for the original angle. Find missing lengths by the Pythagorean Theorem. Then use the appropriate formula, plug in the values by reading them off the triangles and simplify.