1. The Domain and Range of Trig and Inverse Trig Functions

2. You should be familiar with the graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$.

   In this lesson, we will find the domain and range of the basic trig and inverse trig functions.

3. Recall that the domain of a function is the set of possible input values, and the range is the set of possible output values. That is, the domain is the set of possible x-values and the range is the set of possible y-values. In terms of the graph, the domain is the portion of the x-axis that is covered or shadowed by the graph and the range is the portion of the y-axis that is covered or shadowed by the graph.

4. (a) The inputs to the functions sine and cosine are typically thought of as angles on the unit circle, measured in either degrees or radians. Every angle has a sine value and a cosine value, so the domain of both the sine function and the cosine function is everything from $-\infty$ to ∞.

   (b) The range of the sine function is all possible output values. The sine values are found by finding the vertical distance on the unit circle corresponding to an angle. The unit circle has its highest value at 1 (which occurs at $90°$) and its lowest value at -1 (which occurs at $-90°$). The range of the sine function is therefore all numbers between -1 and 1.

   (c) The same applies to the cosine function. The output of the cosine function is a horizontal distance on the unit circle. The largest value occurs at the rightmost point of 1 (which corresponds to the input angle 0) and the smallest value occurs at the leftmost point of -1 (which corresponds to the angle $180°$ or π radians). Therefore the range of the cosine function is also all numbers between -1 and 1.

   (d) The tangent function likewise is defined for all angles, except that the denominator cannot be 0. The denominator of the tangent function is 0 when the point on the unit circle is at the top or the bottom, that is, at $\frac{\pi}{2}$ or $\frac{3\pi}{2}$, or the corresponding angles found by going full circles from those two angles.

   (e) Tangent values can be arbitrarily large. If the angle is slightly less than $90°$, the sine value is 0.99-something and the cosine value is 0.00-something, yielding a very large tangent value. The range of the tangent function is all real numbers.

   (f) For the inverse functions, the inputs and outputs are reversed. The inputs of an inverse trig function are numbers and the outputs are angles. What are the possible inputs for inverse sine and inverse cosine? They are the possible outputs of sine and cosine, namely all numbers between -1 and 1.

   (g) It is tempting to think the range of the inverse sine function is the domain of the sine function for the same reason, but there is a problem. There is more than one angle that has the sine value of $\frac{1}{2}$. We need to decide between $\frac{\pi}{6}$, $\frac{5\pi}{6}$ and all the other angles whose sine is $\frac{1}{2}$. For the inverse sine function, we pick the angles on the right half of the circle, namely the angles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

   (h) For the inverse cosine function, we pick the angles on the top half of the unit circle, the angles from 0 to π.

(i) Since the range of the tangent function is all real numbers, the domain of the inverse tangent is all real numbers

(j) Similar to the sine function, the tangent function runs through its complete list of possible values on the right side of the unit circle, so the range of the inverse tangent function is all angles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

(k) Note that the tangent is undefined at $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, so the range does not include those values.

5. The domain and range can also be found by analyzing the graph of a function. The graphs of sine and cosine go forever to the left and to the right, so the domains of each are all angles. The waves oscillate between -1 and 1, shown here bounded by the green lines, which bound the range between -1 and 1.

6. The tangent graph goes infinitely far below and above the y-axis, so the range is all real numbers.

7. The inverse sine graph looks like this. It begins at the bottom left at the point $(-1, -\frac{\pi}{2})$. It cant go below $-\frac{\pi}{2}$ since if it did, it would start bending back to the right causing the graph to fail the vertical line test. It likewise ends at the point $(1, \frac{\pi}{2})$. The domain goes from -1 to 1, and the range from $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

8. Similarly, the domain of the inverse cosine function goes from -1 to 1 and the range from 0 to π.

9. The domain of the inverse tangent function is all real numbers and the range is from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

10. To recap: Here are the domains and ranges of the basic trig and inverse trig functions.