1. Combining Trig Functions and Inverse Trig Functions - Part I
2. You should be familiar with the six trig functions and inverse trig functions

   In this lesson, we will find values when the two are combined.
3. (a) We will now try to find the exact value of expressions involving both ordinary trig functions, like sine and cosine along with their inverses: Here is an example. Before we get into the specifics of finding the exact answer, we should first understand whether our final answer is going to be a number or an angle. The $\frac{1}{2}$ is a number which represents the length of one side of a right triangle,

   (b) and the inverse cosine function inside the parentheses asks you to find the angle whose cosine is $\frac{1}{2}$. Therefore, the object inside the parentheses is an angle.
4. You are then asked to find the sin of the angle, which will be a number.
5. We can solve this problem directly: using the unit circle, we know that $\frac{\pi}{3}$ is the angle whose cosine is $\frac{1}{2}$.
6. And that $\sin\frac{\pi}{3}$ is $\frac{\sqrt{3}}{2}$.
7. (a) Here is a similar problem: Again, before trying to find the specific answer, it is important to understand which objects are angles and which are numbers. Again in this problem, we begin with a number,

   (b) which is the length of a side of a triangle. The inverse cosine function finds the angle,
8. and then we are asked for the sine of the angle, which is a number, specifically, the height of the triangle.
9. Inside the parentheses, we have the angle whose cosine is $\frac{2}{3}$. Using the unit circle won't help because we dont know the angle whose cosine is $\frac{2}{3}$. However, we can find the value of $\sin\theta$ when we know the value of $\cos\theta$ using the trig version of the Pythagorean Theorem.
10. Here is the problem restated.
11. (a) We use the Pythagorean Theorem

    (b) and solve

    (c) for $\sin\theta$

    (d) The sine value is $\frac{\sqrt{5}}{3}$. We picked the positive value because we are in the first quadrant.
12. (a) We can also do this problem by using the geometric definition to label the triangle

    (b) and use the Pythagorean Theorem to find the value.

13. (a) Here's a similar problem: This time, the cosine value is negative. But the lengths of sides of a triangle can't be negative. We can still do this problem. The cosine values are negative in the second quadrant, so this is the same triangle as in the previous problem,

    (b) You can label the sides of the triangle with positive lengths, and then recognize whether your final answer is positive or negative by noting which quadrant you are in.

    (c) On the other hand, you may wish to include the negative sign on the length of the sides of the triangle as a reminder.

14. (a) Here's an easy one. You begin with a number which is the sine of an angle. The angle is the object in the parentheses, specifically, the angle is the angle whose sine is .276. You are then asked to find the sine of that angle. We label the height of the triangle.

    (b) And then are asked to find the height of the triangle, which is .276

15. (a) Here's one in the opposite direction. We begin with the angle, find the sine of the angle to get a number, and then find the angle that has that number as its cosine.

    (b) $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$, and the angle whose cosine is $\frac{\sqrt{3}}{2} = \frac{\pi}{6}$.

    (c)

16. (a) Working with the tangent function is not much more difficult. We can use the geometry definition to label a triangle

    (b) and read off the appropriate value.

17. Likewise, reading the tangent value from the triangle is not difficult.

18. Secant, cosecant and cotangent also require little extra work. To do a secant problem, do the equivalent cosine problem first, then take the reciprocal.

19. Similarly, if you are given information about the secant, cosecant or cotangent, change it into information about the reciprocal function.

20. To recap: Inverse trig functions give information about the lengths of sides of a triangle, from which you can label a triangle. The trig version of the Pythagoream Theorem allows you to find the missing side. Values of the other trig functions can be read from the triangle.