1. Angle Sum Formulas

2. You should be familiar with techniques used to derive the six angle sum formulas. You should be familiar with inverse trig functions and should be able to simplify fractions containing square roots, including the technique of rationalizing the denominator.

   In this lesson, you will use the formulas to find specific values.

3. Here are the formulas for find the sine, cosine and tangent when adding or subtracting angles. Note that on the left hand side, you are asked to add or subtract the angles, and then find the value of a trig function. On the right hand side, you have three or four numbers found from trig functions that are then added, subtracted, multiplied and divided to get the answer.

4. Before we work some problems, lets ask whether or not we need these formulas at all. Could we compute $\sin(A+B)$ directly? In order to do so, you would need to know angle A and angle B, so that you could add the angles. Maybe you are given information about the angles, in this case, the two angles add to $60°$, and we could find $\sin 60°$ from the unit circle.

5. But you also may be given information about the sine, cosine or tangent of the two angles, and then you would have to use the inverse trig functions to find the angles themselves, before you could add. The formulas will give us a much more direct way to get the answer.

6. (a) Even if you know the two angles and can add them, you may not know the value of the sine unless the angle is one of the special angles on the unit circle.
   (b) In this case, we will have to rely on the angle sum formula.

7. Let's finish this problem. We find $\sin 75°$ by using the angle sum formula. Angle $A = 45°$ and angle $B = 30°$. We look up the values from the until circle, and simplify.

8. (a) Now let's do some examples. In this problem, we are asked to find the $\sin(A+B)$, so we will use $\sin(A+B)$ formula, which requires us to know 4 values. The sine and cosine of angle A, and the sine and cosine of angle B. In this case we aree only given $\sin A$ and $\cos B$ and will need to find the other two values.
   (b) We begin by labelling triangles and finding the remaining side by using the Pythagorean Theorem.
   (c) Note that angle A is given as a first quadrant angle and therefore we use positive side lengths for all the sides, angle B is a fourth quadrant angle, so the cosine of angle B will be a positive number, but the sine of angle B will be a negative number. Once we have labelled the triangles, we can read the appropriate values from the triangle...
   (d) finding the sine and cosine of angle A and the sine and cosine of angle B. We are now ready to use the formula for the sine of angle A plus B, which calls for four numbers which we have already found.
   (e) We plug those four numbers into the formula
   (f) and simplify.

9. (a) Here is a similar example. We may be given information about the triangles using inverse trig functions. In this problem, we have the inverse sine of $3/4$ which is the angle whose sine is $3/4$, and the inverse cosine of $2/3$, which is the angle whose cosine is $2/3$.
   (b) We think of this problem similar to the previous example. We think of the angle A as the angle whose sine is $3/4$ and the angle B as the angle whose cosine is $2/3$, so that we are asked to find the sine of A minus B
   (c) We can label two triangles
   (d) Find the missing side by the Pythagorean Theorem

       We then need to decide which quadrant each of these angles is in, so that we can decide whether the values are positive or negative. Recall that the inverse sine function is always chosen to be on the right half of the unit circle and since this is the angle whose sine is positive $3/4$, that angle should be in the first quadrant, similarly, the inverse cosine function is chosen to be on the top half of the unit circle, in quadrants 1 or 2, and since this angle has a cosine value that is positive, that should also be in the first quadrant. Both of these angles are first quadrant angles.
   (e) Once we have the triangles labelled, we read the values of sine and cosine from the triangles.
   (f) Use the appropriate formula
   (g) Plug in the values and simplify.

10. (a) The tangent angle sum formula is similar.
    (b) First we label triangles and use the Pythagorean Theorem to find the missing sides.
    (c) Next, we look up the values we will need to plug into the formula.
    (d) Plug in those values and simplify. Let's go through the simplification slowly.

11. (a) First, we find a common denominator, which in this case is $2\sqrt{7}$ and rewrtie all the fractions in terms of this denominator. Next, we multiply the overall numerator and denominator by $2\sqrt{7}$
    (b) We rationalize the denominator by multiplying by the conjugate.
    (c) In the numerator, we FOIL. The denominator is the difference of squares pattern. We combine like terms in the numerator. Finally, we can move the negative sign from the denominator.

12. To recap: In most cases, you will be given the value of a single trig function for two different angles and be asked to find the value of a trig function with the angles added or subtracted. To do so, draw and label a triangle for both of the original angles. Find missing lengths by the Pythagorean Theorem. Then use the appropriate formula, plug in the values by reading them off the triangles and simplify.