Solving Triangles Using the Law of Sines - Part II
Preliminaries and Objectives

Preliminaries:
  • Law of Sines

Objectives:
  • Given three parts of a triangle (SSA), find the missing three parts. There may be two possibilities.
Example 1

\[ \frac{\sin 49^\circ}{10} = \frac{\sin C}{12} \Rightarrow \sin C \approx 0.90565 \]

\[ C \approx 64.91^\circ \text{ OR } 115.09^\circ \]
Example 1

\[ A = 49^\circ \quad C = 64.91^\circ \]

\[ B = 66.09^\circ \]

\[
\frac{\sin 66.09^\circ}{b} = \frac{\sin 49^\circ}{10}
\]

\[ \Rightarrow b \approx 12.113 \]

\[ A = 49^\circ \quad C = 115.09^\circ \]

\[ B = 15.91^\circ \]

\[
\frac{\sin 15.91^\circ}{b} = \frac{\sin 49^\circ}{10}
\]

\[ \Rightarrow b \approx 3.632 \]
Example 2

\[ a = 13 \quad c = 12 \quad b \approx 17.199 \]

\[ A = 49^\circ \quad C = 44.16^\circ \]
\[ B = 86.84^\circ \]
\[ \frac{\sin 86.84^\circ}{b} = \frac{\sin 49^\circ}{13} \]
\[ \Rightarrow b \approx 17.199 \]

\[ A = 49^\circ \quad C = 135.84^\circ \]
\[ B = 180^\circ - 49^\circ - 135.84^\circ = -4.84^\circ \]

But we can’t have a negative angle in a triangle!
Example 3

\[
\sin 49^\circ = \frac{\sin C}{\frac{12}{5}} \implies \sin C \approx 1.811
\]
Given two sides and an angle opposite one of the two sides

- Try to find the angle opposite the second side by using the Law of Sines. If you are taking the inverse sine of a number bigger than 1, you will get no solution, otherwise
- Your calculator will give you one possible angle $\theta$, the second possible angle is $180^\circ - \theta$.
- Find the third angle. If the sum of the first two angles is more than $180^\circ$, then there will only be one solution.
- Find the third side of all possible triangles using the Law of Sines.