Solving Triangles Using the Law of Sines - Part I

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Preliminaries and Objectives

Preliminaries:

- Geometric definition of the sine function.
- Geometric proofs that triangles are congruent (ASA, AAS, SSS, SAS)

Objectives:

- Derive the Law of Sines
- Given three parts of a triangle (ASA or ASA), find the missing three parts.

Law of Sines

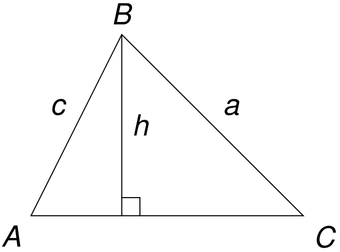
Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Sines

$$\sin C = \frac{h}{a} \quad \Rightarrow \quad h = a \cdot \sin C$$

$$\sin A = \frac{h}{c} \quad \Rightarrow \quad h = c \cdot \sin A$$



Law of Sines

$$\sin C = \frac{h}{a} \Rightarrow h = a \cdot \sin C$$

$$\sin A = \frac{h}{c} \Rightarrow h = c \cdot \sin A$$

$$a \cdot \sin C = c \cdot \sin A \qquad \Rightarrow \qquad \frac{\sin A}{a} = \frac{\sin C}{c}$$

Law of Sines

$$a \cdot \sin C = c \cdot \sin A \quad \Rightarrow \quad \frac{\sin A}{a} = \frac{\sin C}{c}$$

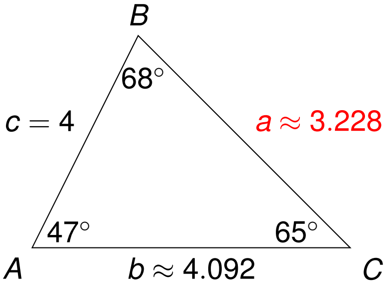
$$a \cdot \sin B = b \cdot \sin A \quad \Rightarrow \quad \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$b \cdot \sin C = c \cdot \sin B \quad \Rightarrow \quad \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

ASA and AAS triangles



$$C = 180^\circ - 68^\circ - 47^\circ = 65^\circ$$

$$\frac{\sin 68^\circ}{b} = \frac{\sin 65^\circ}{4} \Rightarrow b \approx 4.092$$

$$\frac{\sin 47^\circ}{a} = \frac{\sin 65^\circ}{4} \Rightarrow a \approx 3.228$$

Recap

Given two angles and one side

- Find the third angle by summing to 180°
- Find the missing sides by using Law of Sines