Solving Triangles Using the Law of Cosines
Preliminaries and Objectives

Preliminaries:

- The \( \cos \) and inverse \( \cos \) functions
- Law of Sines
- Geometric proofs of congruent triangles

Objectives:

- Given three parts of a triangle (SAS or SSS), find the missing three parts.
Law of Cosines

\[ c^2 = a^2 + b^2 - 2ab \cos C \]

If \( C = 90^\circ \) \( \Rightarrow \cos C = 0 \), then \( c^2 = a^2 + b^2 \)
Law of Cosines

\[ c^2 = a^2 + b^2 - 2ab \cos C \]

\[ b^2 = a^2 + c^2 - 2ac \cos B \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ \frac{\sin 29^\circ}{4.855} = \frac{\sin A}{9} \Rightarrow \sin A \approx 0.8988 \]

\[ \Rightarrow A \approx 64^\circ \]

\[ \Rightarrow B \approx 180^\circ - 29^\circ - 64^\circ = 87^\circ \]
Solving Triangles Using the Law of Cosines

\[ \frac{\sin A}{6} = \frac{\sin 78.46^\circ}{7} \Rightarrow A \approx 57.12^\circ \]

\[ C = 180^\circ - 78.46^\circ - 57.12^\circ = 44.42^\circ \]
The Shortest Distance Between Two Points is a Straight Line

\[ c = 3 \quad a = 4 \]

\[ \cos C \approx 1.109 \approx \cos C \]

Not possible since \(-1 \leq \cos C \leq 1\)
Recap

• SAS - Use Law of Cosines to find the third side, then use Law of Sines to find a second angle that is not the largest angle.
• SSS - Use Law of Cosines to find the largest angle, then use Law of Sines to find a second angle.
• ASA, AAS, SSA - Use the Law of Sines