

Solving Triangles Using the Law of Cosines



Preliminaries and Objectives

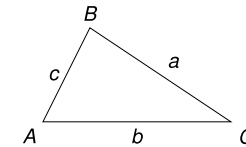
Preliminaries:

- The \cos and inverse \cos functions
- Law of Sines
- Geometric proofs of congruent triangles

Objectives:

- Given three parts of a triangle (SAS or SSS), find the missing three parts.

Law of Cosines



Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

If $C = 90^\circ \Rightarrow \cos C = 0$, then $c^2 = a^2 + b^2$

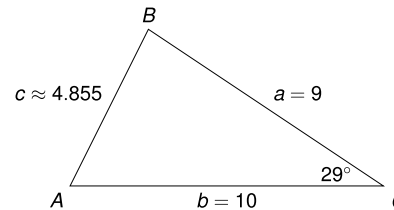
Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

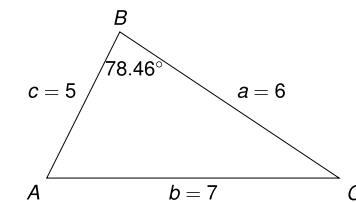
$$a^2 = b^2 + c^2 - 2bc \cos A$$

SAS



$$\begin{aligned} \frac{\sin 29^\circ}{4.855} &= \frac{\sin A}{9} \Rightarrow \sin A \approx .8988 \\ &\Rightarrow A \approx 64^\circ \\ &\Rightarrow B \approx 180^\circ - 29^\circ - 64^\circ = 87^\circ \end{aligned}$$

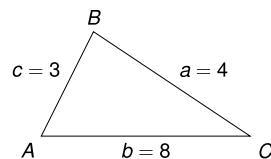
SSS



$$\frac{\sin A}{6} = \frac{\sin 78.46^\circ}{7} \Rightarrow A \approx 57.12^\circ$$

$$C = 180^\circ - 78.46^\circ - 57.12 = 44.42^\circ$$

The Shortest Distance Between Two Points is a Straight Line



$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$3^2 = 4^2 + 8^2 - 2(4)(8) \cos C$$

$$\Rightarrow 1.109 \approx \cos C$$

Not possible since $-1 \leq \cos C \leq 1$

Recap

- SAS - Use Law of Cosines to find the third side, then use Law of Sines to find a second angle that is not the largest angle.
- SSS - Use Law of Cosines to find the largest angle, then use Law of Sines to find a second angle.
- ASA, AAS, SSA - Use the Law of Sines