Solving Triangles Using the Law of Cosines

Preliminaries and Objectives

Preliminaries:
- The cos and inverse cos functions
- Law of Sines
- Geometric proofs of congruent triangles

Objectives:
- Given three parts of a triangle (SAS or SSS), find the missing three parts.

Law of Cosines

\[ c^2 = a^2 + b^2 - 2ab \cos C \]

If \( C = 90^\circ \), \( \cos C = 0 \), then \( c^2 = a^2 + b^2 \)

SAS

\[ c = 4.855 \]
\[ a = 9 \]
\[ b = 10 \]
\[ \theta = 29^\circ \]

\[ \sin 29^\circ = \frac{a}{c} \Rightarrow \sin A = 0.8988 \]
\[ \Rightarrow A \approx 64^\circ \]
\[ \Rightarrow B = 180^\circ - 29^\circ - 64^\circ = 87^\circ \]

SSS

\[ c = 5 \]
\[ a = 6 \]
\[ b = 7 \]
\[ \theta = 78.46^\circ \]

\[ \sin A = \frac{\sin 78.46^\circ}{c} \Rightarrow A \approx 57.12^\circ \]
\[ C = 180^\circ - 78.46^\circ - 57.12^\circ = 44.42^\circ \]

The Shortest Distance Between Two Points is a Straight Line

Recap

- SAS - Use Law of Cosines to find the third side, then use Law of Sines to find a second angle that is not the largest angle.
- SSS - Use Law of Cosines to find the largest angle, then use Law of Sines to find a second angle.
- ASA, AAS, SSA - Use the Law of Sines