Solving Triangles

Preliminaries and Objectives

Preliminaries:
- Law of Sines
- Law of Cosines

Objectives:
- Given three parts of a triangle, find the missing three parts.

AAA

ASA or AAS

Given two angles and one side
- Find the third angle by adding to 180°
- Find a second side using the Law of Sines. You will get the correct approximate answer from your calculator.
- Find the third side using the Law of Sines. Beware of using a rounded off answer to find a second rounded off answer. The rounding errors will compound.
### SAS

Given two sides and an angle opposite one of the two sides

- Try to find the angle opposite the second side by using the Law of Sines. If you are taking the inverse sine of a number bigger than 1, you will get no solution, otherwise
- Your calculator will give you one possible angle $\theta$, the second possible angle is $180^\circ - \theta$.
- Find the third angle. If the sum of the first two angles is more than $180^\circ$, then there will only be one solution.
- Find the third side of all possible triangles using the Law of Sines.

### SSA

Given two sides and an angle opposite one of the two sides

$$
\frac{\sin C}{3} = \frac{\sin 32^\circ}{5.683} \Rightarrow \sin C \approx 0.27975
$$

$$
\Rightarrow C \approx 16.2^\circ \Rightarrow A \approx 131.8^\circ
$$

### SSS

Given three sides of a triangle

$$8^2 = 3^2 + 6^2 - 2(3)(6) \cos A \Rightarrow \cos A = -\frac{19}{36}
$$

$$
\Rightarrow A \approx 121.9^\circ \quad B \approx 39.5^\circ \quad C \approx 18.6^\circ
$$