Solving Triangles Using the Law of Cosines

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Preliminaries and Objectives

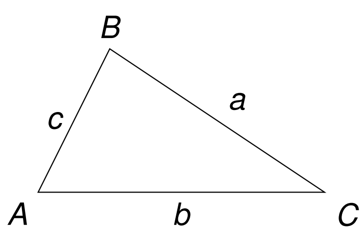
Preliminaries:

- The *cos* and inverse *cos* functions
- Law of Sines
- Geometric proofs of congruent triangles

Objectives:

- Given three parts of a triangle (SAS or SSS), find the missing three parts.

Law of Cosines



Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

If $C = 90^\circ \Rightarrow \cos C = 0$, then $c^2 = a^2 + b^2$

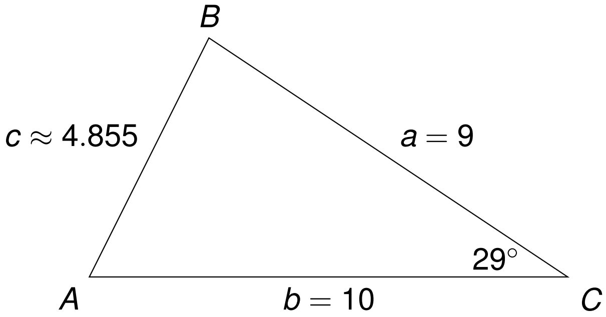
Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

SAS

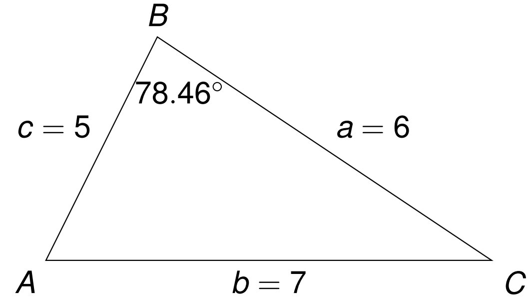


$$\frac{\sin 29°}{4.855} = \frac{\sin A}{9} \Rightarrow \sin A \approx .8988$$

$$\Rightarrow A \approx 64°$$

$$\Rightarrow B \approx 180° - 29° - 64° = 87°$$

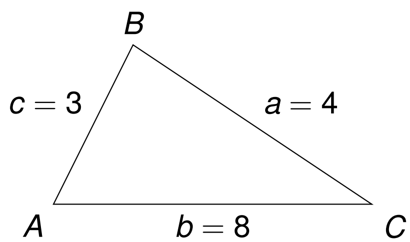
SSS



$$\frac{\sin A}{6} = \frac{\sin 78.46°}{7} \Rightarrow A \approx 57.12°$$

$$C = 180° - 78.46° - 57.12 = 44.42°$$

The Shortest Distance Between Two Points is a Straight Line



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$3^2 = 4^2 + 8^2 - 2(4)(8)\cos C$$

$$\Rightarrow 1.109 \approx \cos C$$

Not possible since $-1 \leq \cos C \leq 1$

Recap

- SAS - Use Law of Cosines to find the third side, then use Law of Sines to find a second angle that is not the largest angle.
- SSS - Use Law of Cosines to find the largest angle, then use Law of Sines to find a second angle.
- ASA, AAS, SSA - Use the Law of Sines