1. Solving Triangles Using the Law of Sines - Part II

2. You should be familiar with the Law of Sines and how it can be used to find the missing parts of triangles in the case when you are given the measure of two angles and the length of one side (ASA and AAS).

   In this lesson, we examine the ambiguous case Side-Side-Angle, which may give two possible answers

3. (a) Here is an example of the type of problem we wish to solve. We are given the lengths of two sides of a triangle and one angle, which is not between the two sides. Note that there is no guarantee that side $a$ does down and to the right from vertex $B$, it may go down and to the left.

      (Insert Animation)

      In this animation, we are given the angle formed by the blue and black sides, which will stay fixed. We are also given the lengths of the blue side and the red side. We will need to find the angle formed by the blue and red sides, the angle formed by the red and black sides, and the length of the black side. We will swing the red side back and forth until it touches the black side, this will determine the two angles involving the red side and the necessary length of the black side. There are three cases. Sometimes the side opposite the given angle is too small, and will not reach the far side. In this case, there will be no solution. Sometimes the opposite side is so long, that we will get one solution on the right, but miss the black side on the left. Sometimes the opposite side is just the right length so that we get two solutions. One solution has an angle less than 90 degrees where the red side meets the black side. The other solution has a an angle larger than 90 degrees where the red side meets the black side.

      (End Animation)

      Returning to our example, we can use the Law of Sines to find angle $C$, with one bit of caution

   (b) the inverse sine function gives you the angle in the FIRST quadrant with the appropriate sine value.

   (c) There is also an angle in the second quadrant, with the same sine value. To get the second angle, subtract the first quadrant angle from $180°$.

   (d) From here, we can solve both triangles, first by finding the third angle

4. (a) Then finding the third side using the Law of Sines.

   (b) .

5. (a) In some cases, the second quadrant angle will be too large, and only the first quadrant angle will work. Here's an example

   (b) We use the Law of Sines to find the two possible angles as usual, your calculator will give the first quadrant angle of $44.16°$, we find the second quadrant angle by subtracting the first quadrant angle from $180°$.

   (c) In this case, the second angle was too large, making the sum of angle $A$ and angle $C$ already over $180°$. Therefore, only the first triangle yields a solution.

6. Here's a third example, in this case, the angle $C$ has a sine value of 1.811, but we know the output of the sine function is always between -1 and 1, so there is no such angle that works and we get no solution. This is the case where the side with length 5 is too short to reach the base of the triangle.

7. To recap, given two sides of a triangle and the angle opposite one of them, try to find the angle opposite the other using the Law of Sines. If the sine value is larger than one, you will get no solution. If you do get an angle when you take the inverse sine, then also find a second quadrant angle and test the two possible triangles. Find the third angle by making the sum of the angles add to $180°$. When using the first quadrant angle, you are guaranteed to get a solution. When using the second quadrant angle, the sum of the first two angles may already be more than $180°$, in which case, you will only have the one solution. You may also get a second solution from the second quadrant angle.