

1. Solving Triangles by Using the Law of Sines - Part I

2. You should be familiar with the geometric definition of the sine function.

It may be helpful to know something about geometric proofs regarding congruent triangles and the naming of the types of problems, like Angle-Side-Angle.

In this lesson, we will prove the Law of Sines and find the missing parts of certain triangles.

3. The Law of Sines gives a relationship between the measures of two angles of a triangle and the lengths of the sides opposite them.

4. (a) The Law of Sines applies to any triangle, not just right triangles. In this case, triangle ABC is not necessarily a right triangle. To be able to use trig functions like sin, which is defined geometrically using right triangles, we need to connect a vertex to the opposite side with a line segment that is perpendicular to that side. This splits the original triangle into two right triangles. The new interior line segment is called the **altitude** and measures the height of the triangle while the side it is connected to is called the **base**.

(b) We can now derive the Law of Sines. We begin by writing an expression for h based on angle C . $\sin C = \frac{h}{a}$, so $h = a \sin C$.

(c) Similarly, using angle A , $\sin A = \frac{h}{c}$, so $h = c \sin A$.

5. (a) Combining these two equations, we get $a \sin C = c \sin A$.

(b) which can be simplified as follows. This is the Law of Sines

6. (a) Repeating this procedure using the other two pair of angles yields three equations

(b) which are often combined in one line.

7. (a) If we are given the length of one side and the measure of two angles of a triangle (ANY triangle, not just a right triangle), we can find the missing parts.

(b) First, we can find the missing angle, since the angles of a triangle add to 180° .

(c) We can then use the Law of Sines to find the missing sides.

(d) For accuracy, when finding the second answer, in this case side a , it is best to use information which is not rounded off, rather than using an approximate answer, like the length of side b to find a second answer which is rounded off.

8. To recap, given two angles and one side, we can first find the third angle because the three angles add to 180° . We can then find the missing sides using the Law of Sines.