Trigonometry
Activity 2a - Transformations of Functions and their Graphs

By the end of this worksheet, you should be able to describe how the graph of

$$y = A \cdot f(Bx + C) + D$$

differs from the graph of $y = f(x)$, for any **arbitrary** function f. Your goal is to figure out how the parameters A, B, C, and D affect the graph. We use some familiar (or not-so-familiar) functions, like parabolas, exponential functions, logarithmic functions, and reciprocals.

1. Translations

   - Task 1: Graph $y = x^2$, $y = x^2 + 4$ and $y = x^2 - 2$
   - Task 2: Graph $y = x$, $y = x + 3$ and $y = x - 1$
   - Task 3: Graph $y = x^2$, $y = (x - 3)^2$ and $y = (x + 1)^2$
   - What do you notice about the shape of the graphs? the location of the graphs?
   - What do the three tasks have in common? How do they differ?
   - What effect will adding D to a function value have on a graph? What effect will adding C to the input value (x) before applying the function have on the graph?
   - Interpret $y = x - 1$ and $y = (x - 1)$ in two different ways and show that their graphs will be the same.
   - Second task for all groups:
     Graph $y = \dfrac{1}{x}$, $y = \dfrac{1}{(x-3)}$ and $y = \dfrac{1}{x} + 2$
   - If you know what the graph of $y = \sin x$ looks like, can you describe what the graph of $y = (\sin x) + 4$ and $y = \sin(x - \frac{\pi}{4})$ look like?

2. Reflections

   - Task 1: Graph $y = e^x$, $y = e^{-x}$, $y = -e^x$ and $y = -e^{-x}$
   - Task 2: Graph $y = \ln x$, $y = \ln(-x)$, $y = -\ln x$ and $y = -\ln(-x)$
   - Task 3: Graph $y = \sqrt{x}$, $y = \sqrt{-x}$, $y = -\sqrt{x}$ and $y = -\sqrt{-x}$
   - What effect will placing a negative sign in front of the function value do to the graph? What effect will placing a negative sign on the input value before applying the function have on the graph?
   - Second task for all groups: Graph $y = x^2$, $y = -x^2$, $y = (-x)^2$ and $y = -(-x)^2$
   - Why do the graphs of $y = x^2$ and $y = (-x)^2$ look the same? Give two reason, one by simplifying the second equation algebraically, the second by interpreting the effect of the negative sign on the graph.
   - If you know what the graph of $y = \sin x$ looks like, can you describe what the graph of $y = -\sin x$ and $y = \sin(-x)$ look like?

3. Magnifications

   - Task 1: Graph $y = \ln x$, $y = 4\ln x$ and $y = \ln(4x)$ (Make special note of where the graph crosses the x-axis.)
   - Task 2: Graph $y = \sqrt{x}$, $y = 2\sqrt{x}$ and $y = \sqrt{2x}$
   - Task 3: Graph $y = \frac{1}{x}$, $y = \frac{2}{x}$ and $y = \frac{1}{2x}$
   - What effect will multiplying A to a function value have on a graph? What effect will multiplying B to the input value (x) before applying the function have on the graph?
   - If you know what the graph of $y = \sin x$ looks like, can you describe what the graph of $y = A\sin x$ and $y = \sin(Bx)$ look like?

4. Summary

   Given the graph of a function $y = f(x)$ and the transformed graph $y = \pm A \cdot f(\pm Bx + C) + D$:

   - Which things in the transformation affect the graph horizontally (left and right) and which affect the graph vertically (top and bottom)?
   - How does a multiplier effect the graph? a minus sign? a number added?
   - If you know the graph of $y = f(x)$, how can you find the graph of $y = -5f(4x) - 3$? the graph of $y = -5f(4x + 2) - 3$?