## Trigonometry

Activity 3c - Angle Sum Formulas Part II

1. In Part I, we developed the following formulas:

$$
\begin{aligned}
& \cos (A+B)=\cos A \cos B-\sin A \sin B \\
& \cos (A-B)=\cos A \cos B+\sin A \sin B
\end{aligned}
$$

Use the first formula to find a formula for

$$
\cos \left(90^{\circ}+\theta\right)
$$

2. It is also true that

$$
\sin \left(90^{\circ}+\theta\right)=\cos \theta
$$

(This can be a bonus problem to be proved by the students)
3. Let $\theta=M+N$.
(a) Use the formula for $\cos \left(90^{\circ}+\theta\right)$ to show that

$$
\sin (M+N)=-\cos \left(90^{\circ}+M+N\right)
$$

(b) Use the formula for $\cos (A+B)$, letting $A=90^{\circ}+M$ and $B=N$ to write a formula for $-\cos \left(90^{\circ}+M+N\right)$
(c) Simplify the previous expression using the formulas found in problems 1 and 2.
(d) Combine problems 3a, b, c, to derive a formula for $\sin (M+N)$.
4. In problem 3) of Part I, you used a process to convert the formula for $\cos (A-B)$ to a formula for $\cos (A+B)$. Use this same process to convert the formula for $\sin (M+N)$ to a formula for $\sin (M-N)$.

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5. Summarize your results:

- $\cos (A-B)=$
- $\cos (A+B)=$
- $\sin (A-B)=$
- $\sin (A+B)=$

6. Find formulas for $\tan (A-B)$ and $\tan (A+B)$ as follows:
(a) Write $\tan (A-B)$ as $\frac{\sin (A-B)}{\cos (A-B)}$, then use 8) to rewrite in terms of $\sin A, \sin B, \cos A$ and $\cos B$.
(b) Divide both the numerator and denominator by $\cos (A) \cos (B)$ and simplify.
(c) Rewrite in terms of $\tan (A)$ and $\tan (B)$.
(d) Repeat the process for $\tan (A+B)$
(e) Summarize your results.

- $\tan (A-B)=$
- $\tan (A+B)=$

