Common Notation for Vectors

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Preliminaries and Objectives

Preliminaries

- Polar Coordinates
- Converting from rectangular coordinates to polar coordinates
- Converting from polar coordinates to rectangular coordinates

Objectives

- Define the norm and magnitude of a vector and basis vectors
- Convert among the three forms of a vector.

Rectangular to Polar Conversion

Rectangular Form:

$\overrightarrow{v} = \langle a, b \rangle$

Polar Form:

Length = Norm = Magnitude = $||\overrightarrow{v}|| = \sqrt{a^2 + b^2}$

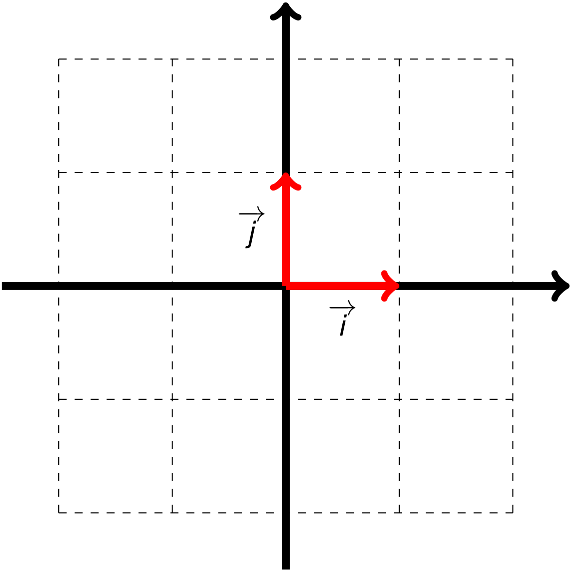
$$\tan\theta = \frac{y}{x}$$

Example: If $\overrightarrow{v} = \langle 4, 3 \rangle$, then

$$||\overrightarrow{v}|| = \sqrt{4^2 + 3^2} = 5$$

$$\tan\theta = \frac{3}{4} \Rightarrow \theta \approx 36.9^\circ$$

Basis Vectors



Three Forms of a Vector

Rectangular Form:

$\overrightarrow{v} = \langle a, b \rangle \qquad a = ||\overrightarrow{v}|| \cos\theta \qquad b = ||\overrightarrow{v}|| \sin\theta$

Basis Vector Form:

$\overrightarrow{v} = a\overrightarrow{i} + b\overrightarrow{j}$

Polar Form:

Length = Norm = Magnitude = $||\overrightarrow{v}|| = \sqrt{a^2 + b^2}$

$$\tan\theta = \frac{y}{x}$$

Example 1

Given $\overrightarrow{v} = \langle -2, 3 \rangle$, find $||\overrightarrow{v}||$, and the direction angle θ.

Solution: $||\overrightarrow{v}|| = \sqrt{(-2)^2 + 3^2} = \sqrt{13} \approx 3.606$

$\tan\theta = -\frac{3}{2}$; $\tan^{-1} -\frac{3}{2} \approx -56.3^\circ$, however $\overrightarrow{v}$ is in Quadrant II, so

$\theta \approx 180 - 56.3^\circ \approx 123.7^\circ$

Example 2

Given $||\overrightarrow{v}|| = 14$ and the direction angle $\theta = 132^\circ$, write $\overrightarrow{v}$ as a linear combination of $\overrightarrow{i}$ and $\overrightarrow{j}$

Solution: $a = 14\cos 132^\circ \approx -9.37$ and $b = 14\sin 132^\circ \approx 10.40$

$$\overrightarrow{v} \approx -9.37\overrightarrow{i} + 10.40\overrightarrow{j}$$

Recap

Rectangular Form:

$\vec{v} = \langle a, b \rangle \qquad a = ||\vec{v}|| \cos\theta \qquad b = ||\vec{v}|| \sin\theta$

Basis Vector Form:

$\vec{v} = a\vec{i} + b\vec{j}$

Polar Form:

Length = Norm = Magnitude = $||\vec{v}|| = \sqrt{a^2 + b^2}$

$\tan\theta = \dfrac{y}{x}$