

Phase Shift



Preliminaries and Objectives

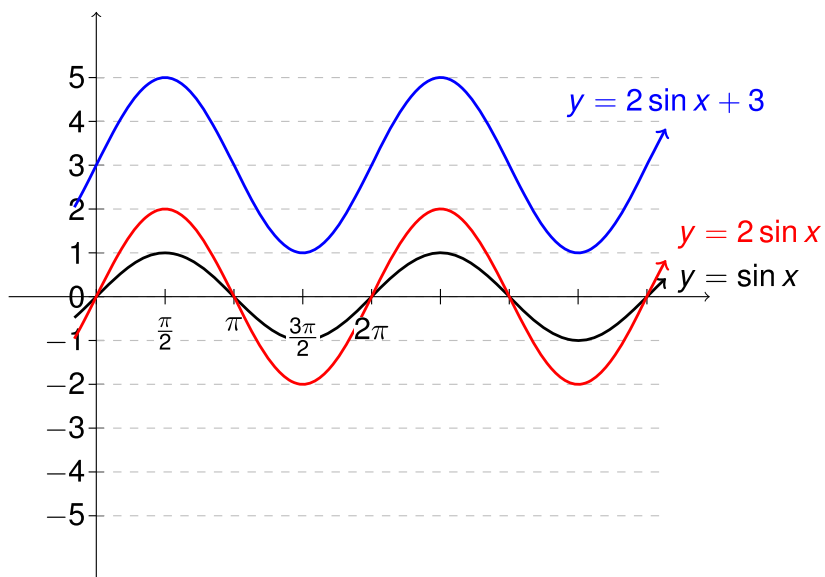
Preliminaries:

- Graphs of $\sin x$ and $\cos x$
- Amplitude
- Period (Wavelength)

Objectives:

- Draw an accurate graph of $y = A\sin(Bx + C) + D$ and $y = A\cos(Bx + C) + D$ over several periods.

$$y = 2 \sin x + 3$$

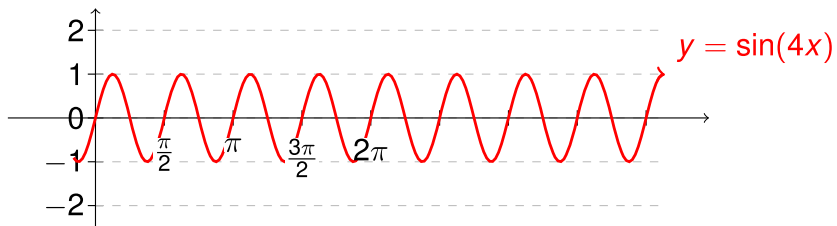


General Forms of Wave Equations

$$y = A \sin(Bx + C) + D$$

$$y = A \cos(Bx + C) + D$$

$$y = \sin\left(4x + \frac{\pi}{4}\right)$$



Phase Shift

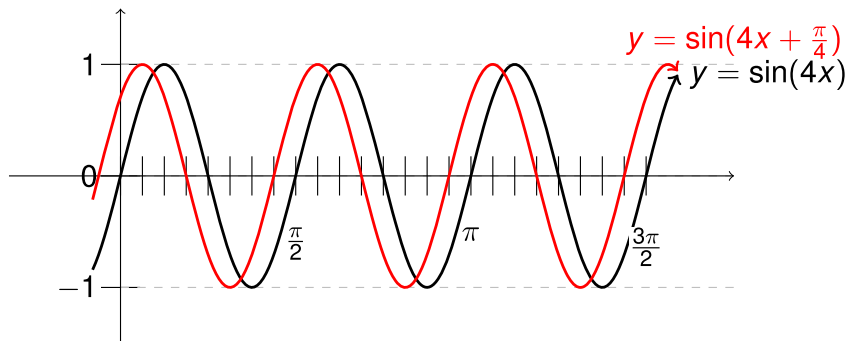
Phase Shift:

$$4x + \frac{\pi}{4} = 0$$

$$\Rightarrow 4x = -\frac{\pi}{4}$$

$$\Rightarrow x = -\frac{\pi}{16}$$

$$y = \sin\left(4x + \frac{\pi}{4}\right)$$



$$\text{Period} = \frac{2\pi}{4} = \frac{\pi}{2} \quad Q = \frac{2\pi}{16} \quad \text{Phase Shift} = -\frac{\pi}{16}$$

General Procedures

Marking the x-axis:

- Find the period
- Find the quarter marks
- Find the phase shift
- Find a common denominator for the quarter marks and phase shift
- Mark the x-axis

Drawing the Graph:

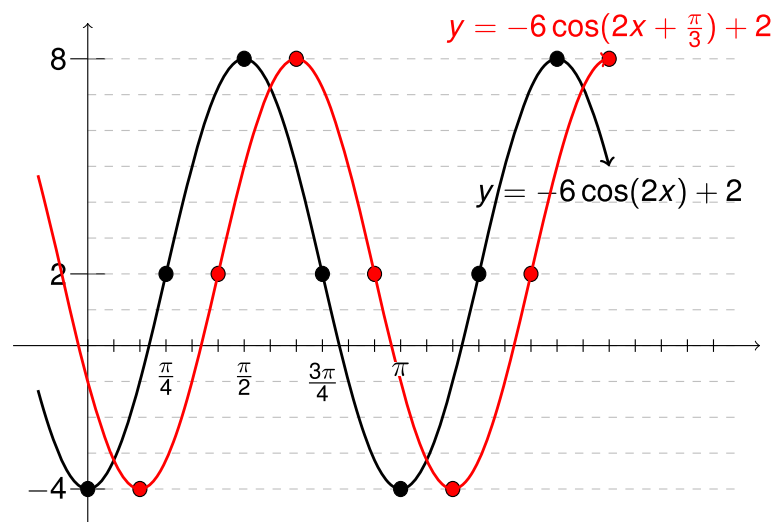
- Mark the reference points (top, middle and bottom of wave) for the unshifted graph and draw the graph.
- Mark the reference points (top, middle and bottom of wave) for the shifted graph and draw the graph.

$$y = -6 \cos(2x - \frac{\pi}{3}) + 2$$

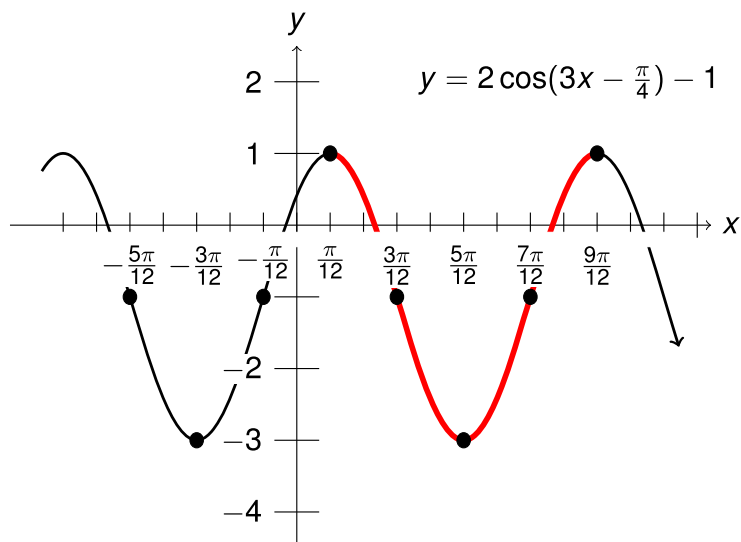
$$y = -6 \cos(2x - \frac{\pi}{3}) + 2$$

- Amplitude = 6
- Vertical Shift = 2
- Centerline = 2, Top = 8, Bottom = -4
- Period = π
- Quarter marks every $\frac{\pi}{4}$
- Phase Shift: $2x - \frac{\pi}{3} = 0 \Rightarrow x = \frac{\pi}{6}$
- Q = $\frac{3\pi}{12}$ Phase Shift = $\frac{2\pi}{12}$

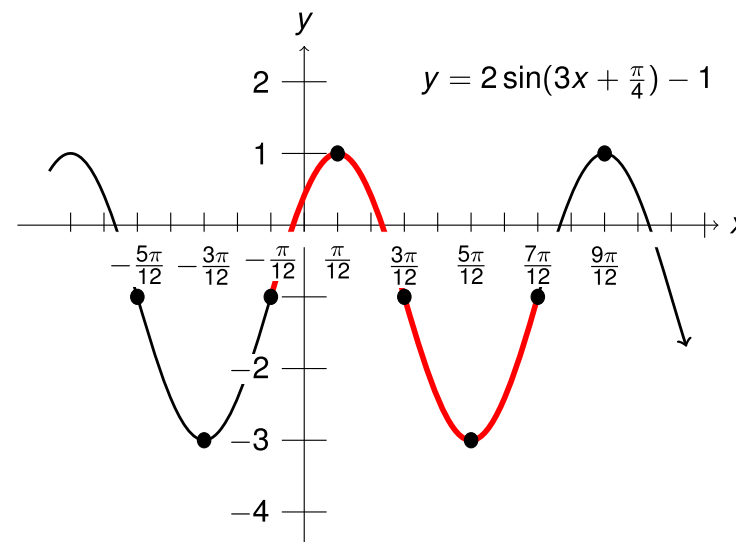
$$y = -6 \cos(2x - \frac{\pi}{3}) + 2$$



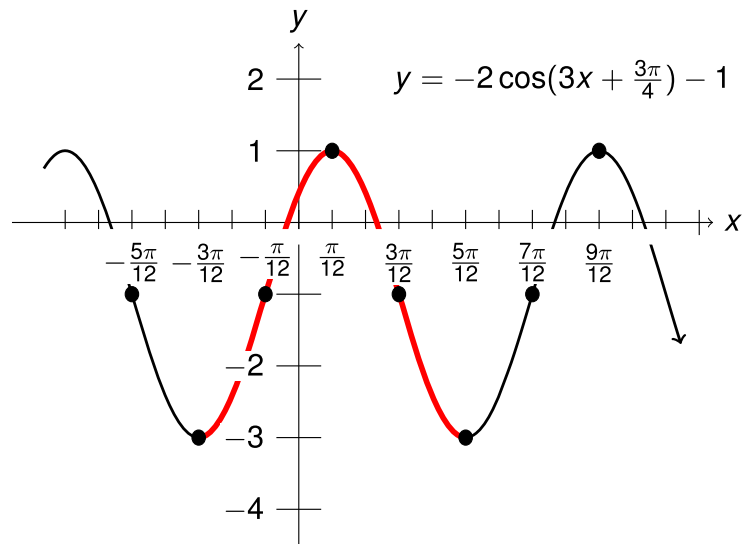
Given the graph, find the equation



Given the graph, find the equation



Given the graph, find the equation



Recap

$$y = A \cos(Bx + C) + D$$

- D determines the y -value of the centerline
- A = amplitude, which is added/subtracted from the centerline value to determine the tops and bottoms of the waves.
- Period = $\frac{2\pi}{B}$, divide by 4 to place quarter markings
- Set $(Bx + C) = 0$ to find phase shift. Mark the x -axis by finding a common denominator between the phase shift and the quarter markings.