

## Common Notation for Vectors



## Preliminaries and Objectives

### Preliminaries

- Polar Coordinates
- Converting from rectangular coordinates to polar coordinates
- Converting from polar coordinates to rectangular coordinates

### Objectives

- Define the norm and magnitude of a vector and basis vectors
- Convert among the three forms of a vector.

## Rectangular to Polar Conversion

Rectangular Form:

$$\vec{v} = \langle a, b \rangle$$

Polar Form:

$$\text{Length} = \text{Norm} = \text{Magnitude} = \|\vec{v}\| = \sqrt{a^2 + b^2}$$

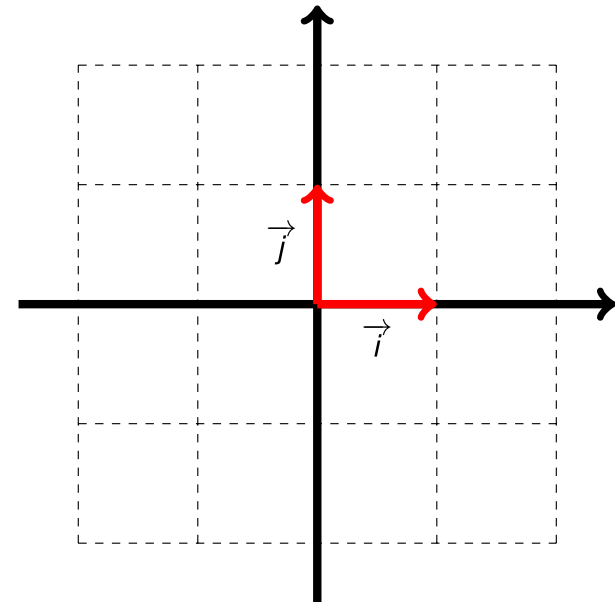
$$\tan \theta = \frac{y}{x}$$

Example: If  $\vec{v} = \langle 4, 3 \rangle$ , then

$$\|\vec{v}\| = \sqrt{4^2 + 3^2} = 5$$

$$\tan \theta = \frac{3}{4} \Rightarrow \theta \approx 36.9^\circ$$

## Basis Vectors



## Three Forms of a Vector

Rectangular Form:

$$\vec{v} = \langle a, b \rangle \quad a = \|\vec{v}\| \cos \theta \quad b = \|\vec{v}\| \sin \theta$$

Basis Vector Form:

$$\vec{v} = a \vec{i} + b \vec{j}$$

Polar Form:

$$\text{Length} = \text{Norm} = \text{Magnitude} = \|\vec{v}\| = \sqrt{a^2 + b^2}$$

$$\tan \theta = \frac{y}{x}$$

## Example 1

Given  $\vec{v} = \langle -2, 3 \rangle$ , find  $\|\vec{v}\|$ , and the direction angle  $\theta$ .

$$\text{Solution: } \|\vec{v}\| = \sqrt{(-2)^2 + 3^2} = \sqrt{13} \approx 3.606$$

$$\tan \theta = -\frac{3}{2}; \tan^{-1} -\frac{3}{2} \approx -56.3^\circ, \text{ however } \vec{v} \text{ is in}$$

Quadrant II, so

$$\theta \approx 180 - 56.3^\circ \approx 123.7^\circ$$

## Example 2

Given  $\|\vec{v}\| = 14$  and the direction angle  $\theta = 132^\circ$ , write  $\vec{v}$  as a linear combination of  $\vec{i}$  and  $\vec{j}$

$$\text{Solution: } a = 14 \cos 132^\circ \approx -9.37 \text{ and } b = 14 \sin 132^\circ \approx 10.40$$

$$\vec{v} \approx -9.37 \vec{i} + 10.40 \vec{j}$$

## Recap

Rectangular Form:

$$\vec{v} = \langle a, b \rangle \quad a = \|\vec{v}\| \cos \theta \quad b = \|\vec{v}\| \sin \theta$$

Basis Vector Form:

$$\vec{v} = a \vec{i} + b \vec{j}$$

Polar Form:

$$\text{Length} = \text{Norm} = \text{Magnitude} = \|\vec{v}\| = \sqrt{a^2 + b^2}$$

$$\tan \theta = \frac{y}{x}$$