

Applications of Vectors



Preliminaries and Objectives

Preliminaries

- Vectors using rectangular coordinates
- Vectors using polar coordinates
- Adding vectors in rectangular coordinates

Objectives

- Adding vectors given in polar coordinates

Conversion between Rectangular and Polar Coordinates

If we know x and y

$$r = \sqrt{x^2 + y^2}$$

$$\tan \theta = \frac{y}{x}$$

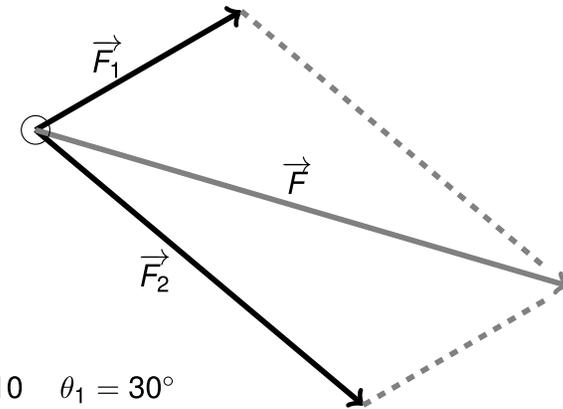
If we know r and θ

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Force Vectors

What is the magnitude of the total force on the object and in which direction is it pointed?



$$\|\vec{F}_1\| = 10 \quad \theta_1 = 30^\circ$$

$$\|\vec{F}_2\| = 18 \quad \theta_2 = 320^\circ$$

Force Vectors

What is the magnitude of the total force on the object and in which direction is it pointed?

$$\|\vec{F}_1\| = 10, \theta_1 = 30^\circ \quad \vec{F}_1 = \langle 8.66, 5.00 \rangle$$

$$\|\vec{F}_2\| = 18, \theta_2 = 320^\circ \quad \vec{F}_2 = \langle 13.79, -11.57 \rangle$$

$$\|\vec{F}_1 + \vec{F}_2\| \approx 23.39 \quad \vec{F}_1 + \vec{F}_2 = \langle 22.45, -6.57 \rangle$$

$$\theta \approx 343.7^\circ$$

Note: $\tan^{-1}\left(\frac{-6.57}{22.45}\right) = -16.3^\circ$; $343.7^\circ = 360^\circ - 16.3^\circ$

Navigational Vectors

A ship is moving through the water at 15 mph, on a heading of 70° . The current is flowing at a rate of 3 mph, on a heading of 310° . What is the speed of the ship and in what direction is it headed?

$$\|\vec{S}\| = 15, \theta_S = 70^\circ \quad \vec{S} = \langle 5.13, 14.10 \rangle$$

$$\|\vec{C}\| = 3, \theta_C = 310^\circ \quad \vec{C} = \langle 1.93, -2.30 \rangle$$

$$\|\vec{S} + \vec{C}\| \approx 13.74 \quad \vec{S} + \vec{C} = \langle 7.06, 11.80 \rangle$$

$$\theta \approx 59.1^\circ$$

The ship is traveling 13.74 mph on a heading of 59.1°

Recap

To add vectors given in polar coordinates:

- 1 Convert to rectangular coordinates
- 2 Add the vectors
- 3 Convert back to polar coordinates