Applications of Vectors

Conversion between Rectangular and Polar Coordinates

If we know $x$ and $y$
\[ r = \sqrt{x^2 + y^2} \]
\[ \tan \theta = \frac{y}{x} \]

If we know $r$ and $\theta$
\[ x = r \cos \theta \]
\[ y = r \sin \theta \]

Preliminaries and Objectives

Preliminaries
- Vectors using rectangular coordinates
- Vectors using polar coordinates
- Adding vectors in rectangular coordinates

Objectives
- Adding vectors given in polar coordinates

Force Vectors

What is the magnitude of the total force on the object and in which direction is it pointed?

\[ \| \vec{F}_1 \| = 10 \quad \theta_1 = 30^\circ \]
\[ \| \vec{F}_2 \| = 18 \quad \theta_2 = 320^\circ \]
Force Vectors

What is the magnitude of the total force on the object and in which direction is it pointed?

\[
\| \vec{F}_1 \| = 10, \, \theta_1 = 30^\circ \\
\vec{F}_1 = \langle 8.66, 5.00 \rangle \\
\| \vec{F}_2 \| = 18, \, \theta_2 = 320^\circ \\
\vec{F}_2 = \langle 13.79, -11.57 \rangle \\
\| \vec{F}_1 + \vec{F}_2 \| \approx 23.39 \\
\vec{F}_1 + \vec{F}_2 = \langle 22.45, -6.57 \rangle \\
\theta \approx 343.7^\circ \\
\]

Note: \( \tan^{-1} \left( \frac{-6.57}{22.45} \right) = -16.3^\circ \); \( 343.7^\circ = 360^\circ - 16.3^\circ \)

Navigational Vectors

A ship is moving through the water at 15 mph, on a heading of 70°. The current is flowing at a rate of 3 mph, on a heading of 310°. What is the speed of the ship and in what direction is it headed?

\[
\| \vec{S} \| = 15, \, \theta_S = 70^\circ \\
\vec{S} = \langle 5.13, 14.10 \rangle \\
\| \vec{C} \| = 3, \, \theta_C = 310^\circ \\
\vec{C} = \langle 1.93, -2.30 \rangle \\
\| \vec{S} + \vec{C} \| \approx 13.74 \\
\vec{S} + \vec{C} = \langle 7.06, 11.80 \rangle \\
\theta \approx 59.1^\circ \\
\]

The ship is traveling 13.74 mph on a heading of 59.1°

Recap

To add vectors given in polar coordinates:

1. Convert to rectangular coordinates
2. Add the vectors
3. Convert back to polar coordinates