1. Polar Coordinates

2. You should be familiar with the Cartesian Coordinate System, also called rectangular coordinates, and with the definitions of sin and cos.

   In this lesson, we will define the Polar Coordinate System, and convert between rectangular and polar coordinates.

3. (a) When a car drives on city streets, we give directions in terms of blocks, and we can find our destination if we know how many blocks to travel north or south, and how many blocks to travel east or west. For instance, you may be directed to go 4 blocks east,

   (b) then 3 blocks north.

   (c) Mathematically, we call this the Cartesian coordinate system. The x-axis measures the distance left and right, and the y-axis measures the distance up and down.

4. (a) However, when navigating in a plane or ship, there are no roads to go by, so we must rely on the compass heading and distance traveled.

   (b) For example, we can travel a distance of 5 in the direction $36.9°$.

   (c) Trigonometry helps us convert between these two systems. If we begin with a point in the Cartesian coordinate system, (the 'city grid' system), we can find the distance from that point to the origin and the direction angle, the so-called 'polar coordinates'.

   If we let r be the distance from the origin to the point (x, y), then we can find r by the Pythagorean Theorem. In this case, $r^2 = 4^2 + 3^2$. We can find the angle $36.9°$ by using the inverse tangent.

5. In general, the Pythagorean Theorem gives $r^2 = x^2 + y^2$, so $r = \sqrt{x^2 + y^2}$. $\tan\theta = \frac{y}{x}$ so $\tan^{-1}\frac{y}{x}$ will give the angle, with one bit of caution. If the angle is in the first quadrant, the inverse tangent will give the correct angle. If x is negative, so that θ is in either the second of third quadrants, a better approach is as follows. Ignore any negative signs on x and y, so that the fraction $\frac{y}{x}$ is positive. The inverse tangent then gives the first quadrant reference angle. You can then add that angle to $180°$ if you are in the third quadrant or subtract that angle from $180°$ if you are in the second quadrant. This approach will also work if θ is in the fourth quadrant.

6. To go in the opposite direction, we wish to find x and y if we know the length and direction. We know $\cos\theta = \frac{x}{r}$ and $\sin\theta = \frac{y}{r}$ so $x = r\cos\theta$ and $y = r\sin\theta$.

7. (a) Here are two examples. You may wish to pause the video to work these out on your own.

   (b) Converting to polar coordinates, note that the ordered pair is in Quadrant II, so the angle should be between $90°$ and $180°$. You calculator will give you an angle of $-18.4°$, which is a fourth quadrant angle. To find the second quadrant angle, subtract $18.4°$ from $180°$.

   (c) The conversion from polar coordinates to rectangular coordinates is straighforward.

8. To recap, to convert from Cartesian coordinates to polar coordinates, find the length by the Pythagorean Theorem and the angle using inverse tangent. To convert from polar coordinates to rectangular coordinates, $x = r\cos\theta$ and $y = r\sin\theta$.