1. Graphs of Waves

2. You should be familiar with the values of sine and cosine between 0 and 2π from the unit circle

   In this lesson, we will draw an accurate graph of the functions $y = \sin x$ and $y = \cos x$

   (Video clip: The graphs of $y = \sin x$ and $y = \cos x$ are known as 'waves'. There is a strong connection between the graph of the sine or cosine function and the waves you see rippling in a body of water, but that is a lesson in physics, so we won't discuss it here.)

3. (a) We will graph $y = \sin x$ and $y = \cos x$ using the special values from the unit circle. We will graph each wave from $0°$ to $360°$, and later from 0 to 2π, and then rely on the fact that the values of these functions repeat.

   (b) We begin by marking the x-axis at $360°$. If you are doing this on your own graph paper, I suggest that you place the mark for $360°$ 24 boxes away from the origin. I'll explain why I picked the number 24 in a moment. The section from $0°$ to $360°$ represents one trip around the unit circle.

   (c) Now let's place some other values on the x-axis. The unit circle is broken into four quadrants, so we will divide the x-axis into four equal parts of $90°$ each. If you are using the graph paper with 24 boxes, then $90°$ will be at the 6th box. Now let's place some points on the graph.

   (d) The largest and smallest values of the sine function are 1 and -1 respectively, so I will place 10 tick marks above and below the axis, with the top and bottom marks at 1 and -1 respectively.

   (e) $\sin 90° = 1$, $\sin 270° = -1$ and $\sin 0°$, $\sin 180°$, and $\sin 360°$ are all 0, so I will place those five points on the graph.

   (f) Next will place points corresponding to the $30 - 60 - 90$ triangle. $30°$ is one-third of $90°$. If you used the graph paper with 24 boxes, you placed $90°$ at the sixth box, so $30°$ will be at the second box.

   (g) $\sin 30° = \frac{1}{2}$, as is $\sin 150°$, while $\sin 210° = -\frac{1}{2}$, $\sin 330° = -\frac{1}{2}$, so let's place those points.

   (h) $\sin 60° = \frac{\sqrt{3}}{2} \approx 0.866$, so we can place the points for $60°$, $120°$, $240°$, and $300°$.

   (i) Finally, we will place the points corresponding to the $45 - 45 - 90$ triangle. If you are using the graph paper with 24 boxes, the mark for $45°$ will be at the third box. This is the smallest gap, the gap between $30°$ and $45°$, is $15°$. $360/15 = 24$, so that's why I chose 24 boxes, if the x-axis is marked every $15°$, we will hit all of our special points.

   (j) $\sin 45° = \frac{\sqrt{2}}{2} \approx 0.707$, so we can place the points for $45°$, $135°$,$225°$, and $315°$.

   (k) We can then connect the dots to form the graph. If you wish to carry on beyond $360°$, you can repeat the same points you have already placed in the same 24 box pattern.

4. (a) We can also do this problem in radians.
   (b) A full circle is 2π
   (c) cut into four equal parts of $\frac{\pi}{2}$
   (d) Each $\frac{\pi}{2}$ is cut into 3 equal parts of $\frac{\pi}{6}$. We can then count by $\frac{\pi}{6}$. $\frac{2\pi}{6}$ is $\frac{\pi}{3}$, $\frac{3\pi}{6}$ is $\frac{\pi}{2}$ which is already labelled, $\frac{4\pi}{6}$ is $\frac{2\pi}{3}$, $\frac{5\pi}{6}$, $\frac{6\pi}{6}$ is π. $\frac{7\pi}{6}$, $\frac{8\pi}{6}$ is $\frac{4\pi}{3}$, $\frac{9\pi}{6}$ is $\frac{3\pi}{2}$, $\frac{10\pi}{6}$ is $\frac{5\pi}{3}$, and finally $\frac{11\pi}{6}$, before reaching $\frac{12\pi}{6}$, which is 2π.
   (e) We can then cut $\frac{\pi}{2}$ in half to get $\frac{\pi}{4}$, $\frac{2\pi}{4}$, which is $\frac{\pi}{2}$, $\frac{3\pi}{4}$, $\frac{4\pi}{4}$ which is π, $\frac{5\pi}{4}$, $\frac{6\pi}{4}$, which is $\frac{3\pi}{2}$, $\frac{7\pi}{4}$, and finally $\frac{8\pi}{4}$, which is 2π.

5. We then can expand the graph by realizing that the graph will repeat every 2π.

6. (a) We can repeat this procedure for $y = \cos x$
   (b) We reach the maximum of 1 at 0° and 360°, and the minimum of -1 at 180°
   (c) The value will be $\pm\frac{\sqrt{3}}{2}$ at 30°, 150°, 210°, 330°
   (d) $\pm\frac{1}{2}$ at 60°, 120°, 240°, 300°
   (e) and $\pm\frac{\sqrt{2}}{2}$ at 45°, 135°, 225°, 315°
   (f) We can then connect the dots

7. and continue to draw multiple waves

8. Here are both graphs together. You may notice that the two graphs looks the same except that the sine wave is the cosine wave shifted to the right. The peaks and valleys repeat the same top-middle-bottom-middle-top pattern, just at different locations.

9. To recap: $y = \sin x$ crosses the x-axis at 0, π, 2π, etc., reaches the top of a wave at $\frac{\pi}{2}$ and again every 2π after that, and reaches the bottom of a wave at $\frac{3\pi}{2}$ and again every 2π after that. $y = \cos x$ crosses the x-axis at the odd multiples of $\frac{\pi}{2}$, starts at the top of a wave at 0, and again every 2π after that, and reaches the bottom of a wave at π and again every 2π after that.