1. Graphs of Other Trig Functions

2. You should be familiar with unit circle values of sine and cosine

   In this lesson, we will draw graphs of $y = \tan x$, $y = \cot x$, $y = \sec x$ and $y = \csc x$.

3. (a) We have seen how to graph $y = \sin x$ and $y = \cos x$. We will now graph the tangent, cotangent, secant and cosecant functions.

   We will begin with $y = \tan x$. Recall that tangent is $\sin / \cos$ and that $\cos x = 0$ at $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. At $\frac{\pi}{2}$, the sine is 1 and the cosine is 0 and 1 divided by zero is undefined. Tangent is also undefined at $-\frac{\pi}{2}$. We will discuss what is happening close to these values later, for now, we will mark those two x-values with dotted lines for asymptotes and draw the graph in between.

   (b) $\sin 0 = 0$ and $\cos 0 = 1$ so $\tan 0 = 0/1 = 0$, so well place a point at $(0, 0)$.

   (c) The sine and cosine of $\frac{\pi}{4}$ are both $\frac{\sqrt{2}}{2}$, and when you divide $\frac{\sqrt{2}}{2}$ by itself, the answer is 1, so well place a point at $(\pi/4,1)$.

   (d) At $-\frac{\pi}{4}$, we get the same result, except that the sine value is negative, while the cosine value remains positive, and therefore $\tan(-\frac{\pi}{4}) = -1$

   Now, back to what is happening near $\frac{\pi}{2}$ and $-\frac{\pi}{2}$. When we have an angle just a little smaller than $\frac{\pi}{2}$, the sine value is almost 1 and the cosine value is almost 0, but both are positive. When we divide, the answer is very large. The graph approaches an asymptote at $\frac{\pi}{2}$, growing very large (and positive) so the graph is going up to infinity.

   The same thing happens at $-\frac{\pi}{2}$, except when we take angles slightly to the right of $-\frac{\pi}{2}$, we are dealing with angles in the fourth quadrant, so the sine is negative while the cosine is positive. When we divide, the answer is large and negative, so we are going down to $-\infty$.

   (e) One branch of the tangent wave looks like this.

   What does the tangent function look like to the right of $\frac{\pi}{2}$? Take a moment to calculate the values of tangent for the angles $\frac{3\pi}{4}$, π and $\frac{5\pi}{4}$. You may wish to pause here to calculate those values.

   The tangent values at $\frac{3\pi}{4}$, π and $\frac{5\pi}{4}$ repeat the same pattern as before, namely the values are $-1, 0$ and 1 respectively. Thus the branch of the tangent function between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ is identical to the branch from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

   (f) Since the unit circle repeats every 2π, so does the graph of tangent and we get a repeating sequence of these branches.

4. (a) Graphing $y = \cot x$ is similar. Recall that cotangent is cosine divided by sine so that the cotangent is undefined when $\sin x = 0$, which is at 0 and π.

   (b) At $\frac{\pi}{2}$, cosine divided by sine is $0/1$ which is zero.

   (c) At $\frac{\pi}{4}$, the cotangent is 1 and at $\frac{3\pi}{4}$, the cotangent is -1.

   (d) The cotangent graph is headed downward toward π.

   (e) And repeats every π, just like the tangent did.

(f) .

5. (a) Cosecant is the reciprocal of sine, so we will use the sine wave to help us graph the cosecant function. The cosecant will be undefined where the sine is zero, which is at 0, π and 2π.
   (b) At $\frac{\pi}{2}$, the reciprocal of 1 is 1, and similarly at $\frac{3\pi}{2}$, the reciprocal of -1 is -1, so we can plot those two points.
   (c) At $\frac{\pi}{6}$, the sine is 1/2, so the cosecant is 2. The same is true at $\frac{5\pi}{6}$.
   (d) The sine is -1/2 at $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$, so the cosecant is -2 at those inputs.
   (e) We can then sketch the cosecant graph from 0 to 2π.
   (f) Since the sine graph repeats every 2π, so does the cosecant graph.

6. (a) The process for secant is similar. You may wish to try it on your own before watching the end of this video.

   We will start with the cosine wave.
   (b) Place asymptotes where the cosine is zero.
   (c) Place the values 1 and -1.
   (d) And the values ± 2 at the $\frac{\pi}{6}$ markings.
   (e) Sketch the graph from 0 to 2π.
   (f) and repeat every 2π.