1. Frequency, Wavelength and Period

2. You should be familiar with the graphs of $y = \sin x$ and $y = \cos x$, as well as $y = \pm A \sin x$ and $y = \pm A \cos x$ and how the amplitude affects the shape of the graph. You should also be familiar with transformations of graphs, and how to stretch graphs vertically and horizontally.

   In this lesson, we will stretch the graph horizontally, changing the distance between peaks of the wave, called **wavelength** or **period**. We will find the period and frequency from the equation and also from the graph of a wave. We will graph waves, given the equation.

3. Recall that a multiplier in front of the sin function will make the graph taller. This stretch factor is called the amplitude.

4. We will now look at how a multiplier to the input variable x will affect the length of the wave.

   Recall that multiplying the input doesn't make make the graph wider, it makes the graph thinner.

5. The graph of $y = \sin x$ goes through one complete wave every 2π. The length of a wave is called the wavelength or the period. We will use the terms wavelength and period interchangibly. The wavelength can be found by measuring the x-distance between the top of one wave and the top of the next. For $y = \sin x$, the period is 2π. We could also measure the wavelength using consecutive low points. We could also measure the wavelength using the middle points.

6. (a) For $y = \sin(2x)$, the wave should be twice as narrow, so that instead of going through a full wave every 2π, it goes through a full wave twice as fast, or every π.
   (b) The wavelength or period is π.

7. The idea of frequency is more intuitive to physicists. The frequency of a wave the is number of waves that are completed in one unit of time, or in this case, one unit on the x-axis. If the wavelength is 2π, then there is one wave every 2π, so the frequency is $\frac{1}{2\pi}$. In this case, $B = 2$, so there are two waves every 2π, or one wave every π, so the frequency is $\frac{1}{\pi}$. Notice that when $B = 2$, the frequency doubled, and the period is cut in half.

8. (a) For $y = \sin(4x)$, the wave should be four times as narrow, so that instead of going through a full wave every 2π, it goes through a full wave four times as fast, or every $\frac{\pi}{2}$.
   (b) The wavelength or period is $\frac{\pi}{2}$. In general, the period is $\frac{2\pi}{B}$.
   (c) The frequency in this example is $\frac{4}{2\pi}$. In general, the frequency is $\frac{B}{2\pi}$.

9. Here are the formulas for period and frequency.

10. (a) To draw a graph of $y = \sin(5x)$, we begin with the period, which is $\frac{2\pi}{5}$
    (b) We will divide $\frac{2\pi}{5}$ into 4 equal parts, each of which is $\frac{2\pi}{20}$ or $\frac{\pi}{10}$. I will call these four values the 'quarter' markings and denote this spacing with the letter 'Q'. We can then make the four quarter markings on the x-axis at $\frac{\pi}{10}$, $\frac{2\pi}{10}$, $\frac{3\pi}{10}$, and $\frac{4\pi}{10}$,
    (c) place points starting at $x = 0$ that are in the middle, top, middle, bottom, and middle. Note that the amplitude is 1.
    (d) And sketch the wave.

11. (a) We can graph a wave where both the amplitude and period have been changed. For $y = -2\cos(3x)$, we will first mark the x-axis. The period is $\frac{2\pi}{3}$.
    (b) which we divide into 4 equal parts of $\frac{2\pi}{12}$, or $\frac{\pi}{6}$.
    (c) The amplitude is 2, so we will place marks up to 2 and down to -2 on the y-axis.
    (d) A standard cosine wave goes from the top to the middle, to the bottom, to the middle to the top, but this cosine wave is flipped top to bottom because of the negative sign, so we will use the pattern bottom, middle, top, middle, bottom.

12. (a) Now let's work backwards. Here is a graph, let's see if we can find the equation. The graph passes through the origin, which is what a sine wave is supposed to do, but it starts out going down, rather than up, so we should begin with a negative sign to flip the graph.
    (b) It goes up to a height of 3 and down to -3, so the amplitude is 3.
    (c) The graph completes its first full wave at 4π, so the period is 4π.
    (d) Thus 4π is $\frac{2\pi}{B}$, making $B = \frac{1}{2}$. The equation of the graph is $y = -3\sin\left(\frac{1}{2}x\right)$.

13. (a) Amplitude and period are measured in the same way for cosine waves. Lets see if we can find the equation for this graph. The graph is at its high point when $x = 0$, this is how we tell the sine wave from the cosine wave. The sine wave passes through the origin, the cosine wave is at its peak at $x = 0$.
    (b) The wave goes up to 2 and down to negative 2, so the amplitude is 2.
    (c) The graph completes a full wave at $\frac{\pi}{3}$, so $\frac{\pi}{3} = \frac{2\pi}{B}$, making $B = 6$. The equation of the graph is $y = 2\cos(6x)$.

14. To recap: The period, or wavelength is the x-distance between the tops of two consecutive waves. If you are given the graph, you can measure the period directly. The frequency is the reciprocal of the period. Frequency and period can be found from the equation using the formulas here. To graph a wave from the equation, mark off the y-axis using the amplitude. Find the period using the formula and divide it into four equal parts (Q) to place the quarter markings. Then place the key points and sketch the graph.