1. Common Notation for Vectors

2. You should be familiar with polar coordinates and the conversions between polar coordinates and rectangular coordinates.

   In this lesson, we will define some commonly used terminology and notation and convert among various forms of vectors

3. There are two main ways to specify a vector. The first is by the rectangular (or Cartesian) coordinates, that is, by the ordered pair.

4. The second is to specify the length and direction of the vector.

5. To convert from rectangular (Cartesian) coordinates to polar coordinates, find the length by the Pythagorean Theorem and the angle by the inverse tangent. In vector terminology, the length is also referred to as the norm or the magnitude. The symbol for the length looks like a double absolute value, which is appropriate. For a real number, the absolute value specifies the size of the number, the distance from 0. The plus or minus sign tells us the direction, whether to go left or right on the number line. In two dimensions, the magnitude tells us the size of the vector. We then need to know in which direction the vector is pointing.

6. A third way to describe vectors comes from the desire to decompose vectors into vertical and horizontal components. We give a special name to the vector which goes one unit to the right, that vector is named $\overrightarrow{i}$. Likewise, the vector which goes one unit up is called $\overrightarrow{j}$.

7. Now instead of writing a vector as an ordered pair $\langle a, b\rangle$, we can write it as a sum of its vertical and horizontal components. We can scale the vectors $\overrightarrow{i}$ and $\overrightarrow{j}$ to any length, and achieve any vector by a sum. The basis vector form of a vector is merely the rectangular form rewritten as a sum, rather than as an ordered pair. It is common to say that the basis form of a vector is expressed as a *linear combination* of $\overrightarrow{i}$ and $\overrightarrow{j}$.

8. (a) Here is an example. We are given the rectangular form of the vector, and wish to find the polar form.

   (b) We find the magnitude by the Pythagorean Theorem. We use the inverse tangent to find the angle. Notice that the vector $\overrightarrow{v}$ points into the second quadrant, but the inverse tangent gives a fourth quadrant angle. We can find the corresponding angle in the correct quadrant by finding 180° - 56.3°.

9. (a) In this second example, we are given the magnitude and direction of the vector $\overrightarrow{v}$, and need to convert to the rectangular form.

   (b) The x-coordinate is the length times the cosine of the angle and the y-coordinate is the length times the sine of the angle.

10. To recap: The cos and sin functions allow us to convert from polar form to rectangular form. The Pythagorean Theorem and the inverse tangent function allow us to convert in the other direction. The rectangular form can also be written as the linear combination of $\overrightarrow{i}$ and $\overrightarrow{j}$, which are vectors of length 1 along the x and y-axis respectively.