

1. Amplitude

2. You should be familiar with the graphs of sine and cosine and with transformations of graphs. Specifically, you should know how to make graphs taller or shorter by adjusting the equation.

In this lesson, we will give a definition of the **amplitude** of a wave, find the amplitude from the graph of a wave and write the equation, and also go in the opposite direction, given the equation, graph the wave.

3. (a) Recall the basic sine wave. It has x -intercepts at $0, \pi, 2\pi$ etc. It reaches its high point at $\frac{\pi}{2}$ and its low point at $\frac{3\pi}{2}$. The tops of the waves are at a height of 1, and the bottoms of the waves are at a depth of -1.
- (b) The length of one wave is 2π . The red wave here goes through the pattern ‘middle-top-middle-bottom-middle’ from 0 to 2π . The wave then repeats this pattern from 2π to 4π , and repeats again and again, both to the left and to the right.
- (c) The standard sine wave goes up to a maximum value of 1 and down to a minimum value of -1. The distance from the midline to the top is called the amplitude, so $y = \sin x$ has an amplitude of 1. The amplitude could also be measured by taking half of the distance from the top of the wave to the bottom of the wave.
4. We will now look at ways that we can modify our wave graphs. The first modification will be to multiply the sine value by a constant, in this case A . Recall that a multiplier of the output of the function value will stretch the graph vertically, so that the graph will be taller. In this case, the waves will be taller. The sine wave still goes through the ‘middle-top-middle-bottom-middle’ pattern from 0 to 2π , but the tops of the waves are higher and the bottoms of the waves are lower.
5. (a) How does the graph of $y = 2 \sin x$ compare to the graph of $y = \sin x$? You may wish to pause the video here and draw the wave yourself.
- (b) The waves in $y = 2 \sin x$ go up to 2 and down to -2. They are twice as tall as usual. The amplitude is 2.
- (c) In red is a graph whose waves go up to a height of 5 and down to a depth of -5. Is it a sine wave? Yes, the x -intercepts are the usual $0, \pi, 2\pi$, etc. It repeats every 2π . The wave reaches its peak at $\frac{\pi}{2}$, it is merely a sine wave that is five times as tall as usual. What is its equation?
- (d) The amplitude is 5, so the equation is $y = 5 \sin x$.
- (e) If the multiplier is negative, the wave will be reflected across the x -axis, in other words, the graph will be flipped top-to-bottom. The graph of $y = -\sin x$, is the graph of $y = \sin x$ flipped top-to-bottom, so that from 0 to $\frac{\pi}{2}$, the wave goes down rather than up. We still say the amplitude is 1.
- (f) The graph of $y = -3 \sin x$ will have an amplitude of 3, reach it’s low point of -3 at $\frac{\pi}{2}$, return to 0 at π , reach it’s high point of 3 at $\frac{3\pi}{2}$ and return to 0 at 2π .

6. (a) Given a graph, can we find the equation? First, is this a sine wave? Sine waves begin in the middle when x is 0, therefore this is not a sine wave. Cosine waves begin at the top when $x = 0$, this wave begins at the bottom, so this is a cosine wave that has been reflected across the x -axis, that is, it has been flipped top-to-bottom. This is a negative cosine wave.
- (b) The amplitude is 4,
- (c) therefore the equation is $y = -4 \cos x$.
7. To recap: You should now be able to draw the graph of a sine or cosine wave, multiplied by a constant, perhaps a negative constant. If the amplitude is A , then the peaks have a height of A and the valleys have a depth of $-A$. If you are given the graph of a wave, you should determine if it is a sine wave, or a cosine wave. At $x = 0$, sine waves will be in the middle of the wave at $y = 0$. Cosine waves will be at the top of the wave if positive; at the bottom of the wave if negative. You can then determine the amplitude by measuring the distance from the middle of the wave to the top.