1. Adding and Subtracting Vectors

2. You should be familiar with the Cartesian Coordinate System and with vectors.

   In this lesson, we will define some basic properties of vectors, and learn how to add vectors, subtract vectors and scale vectors, that is, make vectors longer or shorter without changing their direction.

3. (a) We can think of the ordered pair $(4, 3)$ as a point in the plane which is 4 units to the right and 3 units above the origin. In many applications, it is helpful to think of the ordered pair $(4, 3)$ as a motion that is going 4 units right and 3 units up.

   (b) This motion is independent of the starting location. For instance the vector which starts at $(-3, 1)$ and ends at $(1, 4)$ is the same as the vector which starts at the origin and ends at $(4, 3)$ in the following sense: the two red vectors have the same length, and are pointed in the same direction, in other words, they are parallel.

   The vector describes the amount we move from beginning to end. In both of these vectors, we move 4 to the right and 3 up.

4. In general, to find the ordered pair for a vector, find each coordinate by subtracting the beginning coordinate from the ending coordinate.

5. (a) We often are asked to perform two motions, one following the other. Where do we wind up if we first follow the red vector, going right 4 and up 3, then perform the blue vector, going left 2 and up 1?

   (b) Geometrically, we can see the answer by moving the blue vector so that it starts where the red vector ends. If we go 2 left and 1 up from the point $(4, 3)$, we wind up at the point $(2, 4)$.

   (c) We can think of this as the definition of vector addition. When we add the blue vector to the red vector, we get the black vector as the answer. The black vector represents the combined motion of the two individual components.

6. Mathematically, we get the sum of two vectors by adding the coordinates of the individual vectors.

7. The opposite of a vector is a vector of the same length that points in the opposite direction. If we first followed the red vector, then the blue vector, we would return to where we started.

8. (a) To subtract one vector from another, we add the opposite. Geometrically, this turns the subtracted vector around.

   (b) Instead of adding the blue vector which goes 2 left and 1 up, we can subtract the blue vector by going 2 right and one down.

   (c) Subtracting the blue vector from the red vector is equivalent to adding a vector which points in the opposite direction as the original blue vector.

9. We subtract vectors by subtraction coordinates

10. (a) Can we multiply two vectors and divide two vectors? In other fields, such as physics, there are two common vector products called the dot product and the cross product, but we will save those for other lessons. We can however change the size of a vector, that is, change the scale. To make the vector $(4, 3)$ twice as long, all we have to do is double each coordinate.

    (b) Two times the red vector is the blue vector.

11. Mathematically, to scale a vector by a factor of k, multiply each coordinate by k.

12. To recap: To add vectors, add the coordinates. To subtract vectors, subtract the coordinates. To make a vector longer or shorter, multiply each coordinate by the scale factor.