1. The Unit Circle - Part I

2. You should be familiar with the measurement of angles and the Cartesian Coordinate System, including the naming of the four quadrants.

   In this lesson, we will give a trigonometric definition of the functions *sin* and *cos* and find some values of these functions.

3. (a) You may be familiar with the geometric definition of sine and cosine, but the intent of this lecture is to give an alternate definition, called the trigonometric definition. We begin with a circle of radius 1, centered at the origin.

   (b) We will measure angles, whose vertex is at the origin, one side is on the positive side of the x-axis, and for small angles, whose measure is positive but less than $90°$, the other side will be in the first quadrant. The black side will stay fixed, and we will move the red side to get various angles.

   (c) We can then associate a point (that is, an ordered pair) to each angle.

   What we have just described, is a function. The input of the function is the angle, in this case $60°$, and the output of the function is an ordered pair which specifies the location of a point. In this case, the point appears to have an x-coordinate of .5 and a y-coordinate between .8 and .9. As the angle changes, so will the point.

   Unlike the geometric definition, which is valid only for angles between $0°$ and $90°$, there is no need to restrict ourselves to the first quadrant. For instance, the angle can be in the second quadrant. We can have a straight angle that measures $180°$, or the angle can even be bigger than $180°$. Just as a skateboarder can do a '360', we can likewise make a full trip around the circle to arrive at the angle $360°$.

4. Returning to our function: Let's take, for example, the angle $90°$, which will have its variable red side, pointed straight up the y-axis. This angle is associated with the point $(0, 1)$. Our function has an input angle of $90°$ and has as its output, the ordered pair, $(0, 1)$.

5. The straight angle of $180°$ will be associated with the point $(-1, 0)$, so our function has an input angle of $180°$ and an output ordered pair of $(-1, 0)$.

6. If the input is the angle $360°$, the output is $(1, 0)$.

7. (a) As a test of your understanding, find the point associated with the angle $270°$.

   (b) If the input is the angle $270°$, the output is the ordered pair $(0, -1)$.

8. We can also have the angle equal $0°$. In this case, the point will be $(1, 0)$.

9. We can also have the angle go clockwise from the x-axis, rather than counterclockwise. In this case, the angle will be measured as a negative angle. For example, the angle $-90°$ corresponds to the point $(0, -1)$.

10. To recap, we have developed a function, which has as its input, an angle, and the output of the function is a point, that is, an ordered pair on the unit circle. We are now ready to define the cosine function and the sine function. The cosine function is merely the x-coordinate of the ordered pair of our previous function and the sine is the y-coordinate of our previous function. For example, for the angle 180°, the point on the unit circle is $(-1, 0)$, therefore $\cos 180^\circ = -1$, since the x-coordinate of the point is -1, and $\sin 180^\circ = 0$, since the y-coordinate of the point is 0.

11. (a) To test your understanding, try the following problems. You may wish to pause the video at this point in order to work out the answers to these problems.

    (b) .

12. To recap: The angle 0° is associated with the positive side of the x-axis and therefore the point $(1, 0)$. Angles are then measured counterclockwise from the positive side of the x-axis. The cosine of an angle is the x-coordinate of the associated point, and the sine of an angle is the y-coordinate of the associated point.