1. The Unit Circle - Part II

2. You should be familiar with the trigonometric definitions of the functions *sin* and *cos*, that is, sine and cosine are defined as coordinates on a circle of radius 1, called the unit circle. You should also be familiar with the measurement of angles in both radians and degrees. Finally, you should be familiar with the $30-60-90$ triangle and the $45-45-90$ triangle. In this lesson, we will find values for *sin* and *cos* for angles associated with the $30-60-90$ triangle and the $45-45-90$ triangle.

3. To recap, we have developed a function, which has as its input, an angle, which can be measured in either degrees or radians, with the zero angle at the positive side of the x-axis, and positive angles moving counterclockwise from there, and the output of the function is a point, that is, an ordered pair on the unit circle. The cosine of an angle is the x-coordinate of the corresponding point, shown here in blue, and the sine of an angle is the y-coordinate of the corresponding point, shown here in red.

4. (a) To test your understanding, try the following problems. You may wish to pause the video at this point in order to work out the answers to these problems.
   (b) .

5. There are two special right triangles that will allow us to find points on the circle with radius 1, centered at the origin. These two triangles are the isosceles right triangle, otherwise known as the $45-45-90$ triangle, and the $30-60-90$ triangle.

6. The $45-45-90$ triangle has two legs, each with length $\frac{\sqrt{2}}{2}$.

7. We can use this triangle to find the x- and y-coordinates of the points associated with $45°$, $135°$, $225°$, and $315°$. The standard is to always place one leg of the triangle on the x-axis. At $45°$, we are in the first quadrant, so both the x-coordinate and the y-coordinate are positive. That is, both $\cos 45°$ and $\sin 45°$ are positive. Since the leg along the x-axis, which measures the x-distance from the origin, and therefore is the x-coordinate, has length $\frac{\sqrt{2}}{2}$, $\cos 45° = \frac{\sqrt{2}}{2}$. Similarly, the height of the triangle is $\frac{\sqrt{2}}{2}$, so $\sin 45° = \frac{\sqrt{2}}{2}$. We get the same values at $135°$, except we are now moving to the left, rather than the right, so the x-coordinate, that is the cosine, should be a negative value. The sine of $135°$ is still above the centerline, so it is positive. $\cos 135° = -\frac{\sqrt{2}}{2}$ and $\sin 135° = \frac{\sqrt{2}}{2}$. At $225°$, we are in the third quadrant, so both the cosine and sine are negative. At $315°$, the cosine is positive, but the sine is negative. Here are all four angles.

8. We can convert the degree measures to radians.

9. (a) To test your understanding, try these problems.
   (b) .

10. The $30-60-90$ triangle has one leg with length $\frac{1}{2}$ and the other leg with length $\frac{\sqrt{3}}{2}$. This triangle can be oriented in two ways, with the long side on the x-axis and the short side up, or with the short side on the x-axis and the long side up.

11. Let's begin with the long side on the x-axis so the four angles will be $30°$, $30°$ back from $180°$, which is $150°$; $30°$ forward from $180°$, which is $210°$; and $30°$ back from $360°$, which is $330°$. In the first quadrant, the angle is $30°$, and therefore $\cos 30° = \frac{\sqrt{3}}{2}$, and $\sin 30° = \frac{1}{2}$. We can also find cosine and sine values for $30°$ back from $180°$, which is $150°$; $30°$ forward from $180°$, which is $210°$; and $30°$ back from $360°$, which is $330°$. Here are the set of all four angles, where the cosine value is either $\pm\frac{\sqrt{3}}{2}$, and the sine values are either $\pm\frac{1}{2}$.

12. The angles can be measured in radians.

13. (a) And some problems.

    (b) .

14. We can also put the short side on the x-axis to find sine and cosine values for $60°$, $120°$, $240°$, and $300°$. In the first quadrant, the angle at the origin is $60°$, so $\cos 60° = \frac{1}{2}$ and $\sin 60° = \frac{\sqrt{3}}{2}$. Similarly, we can find cosine and sine values for $60°$ back from $180°$, which is $120°$; $60°$ forward from $180°$, which is $240°$; and $60°$ back from $360°$, which is $300°$. Here is the set of all four angles, where the cosine value is $\pm\frac{1}{2}$ and the sine value is $\pm\frac{\sqrt{3}}{2}$.

15. The angles can be measured in radians.

16. (a) And some problems.

    (b) .

17. To recap: Altogether, there are 16 special angles, starting at $0°$, and ending at $360°$, or 2π radians, for which you should be able to find the cosine and sine values.