Pythagorean Theorem - Trig Version
Preliminaries and Objectives

Preliminaries:
- Pythagorean Theorem
- Trig Definition of $\sin \theta$ and $\cos \theta$

Objectives:
- Given $\sin \theta$, find $\cos \theta$
- Given $\cos \theta$, find $\sin \theta$
Pythagorean Theorem

\[ \cos^2 \theta + \sin^2 \theta = 1 \]
Example 1

If \( \cos \theta = \frac{2}{5} \) and \( \frac{3\pi}{2} < \theta < 2\pi \), find \( \sin \theta \).

Solution:

\[
\left( \frac{2}{5} \right)^2 + \sin^2 \theta = 1
\]

\[
\Rightarrow \sin \theta = \pm \frac{\sqrt{21}}{5}
\]

Since \( \theta \) is in quadrant IV, \( \sin \theta = -\frac{\sqrt{21}}{5} \).
Further Examples

If \( \sin \theta = \frac{3}{7} \) and \( \frac{\pi}{2} < \theta < \pi \), find \( \cos \theta \).

If \( \cos \theta = -\frac{2}{3} \) and \( \frac{\pi}{2} < \theta < \pi \), find \( \sin \theta \).
Further Examples

If \( \sin \theta = \frac{3}{7} \) and \( \frac{\pi}{2} < \theta < \pi \), find \( \cos \theta \).

Solution:

\[
\left( \frac{3}{7} \right)^2 + \cos^2 \theta = 1
\]

\[
\cos^2 \theta = 1 - \frac{9}{49} = \frac{40}{49}
\]

\[
\cos \theta = \pm \frac{\sqrt{40}}{7} = \pm \frac{2\sqrt{10}}{7}
\]

\[\Rightarrow \cos \theta = -\frac{2\sqrt{10}}{7} \text{ since } \theta \text{ is in quadrant II.}\]
If \( \cos \theta = -\frac{2}{3} \) and \( \frac{\pi}{2} < \theta < \pi \),

find \( \sin \theta \).

Solution:

\[
\sin^2 \theta + \left( -\frac{2}{3} \right)^2 = 1
\]

\[
\sin^2 \theta = 1 - \frac{4}{9} = \frac{5}{9}
\]

\[
\sin \theta = \pm \frac{\sqrt{5}}{3}
\]

\( \Rightarrow \sin \theta = \frac{\sqrt{5}}{3} \) since \( \theta \) is in quadrant II.
If $\cos \theta = \frac{2}{5}$ and $\theta$ is in quadrant IV, find $\sin \theta$.

Solution:

$$2^2 + b^2 = 5^2$$
$$\Rightarrow b^2 = 25 - 4 = 21$$
$$\Rightarrow b = \pm \sqrt{21}$$

Therefore $\sin \theta = -\frac{\sqrt{21}}{5}$ since $\theta$ is in quadrant IV.
Recap

- Pythagorean Theorem

\[ \sin^2 \theta + \cos^2 \theta = 1 \]

- Information about the quadrant will determine whether answer is positive or negative.

- Can use geometric definition of sine and cosine.