Pythagorean Theorem - Trig Version

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Preliminaries and Objectives

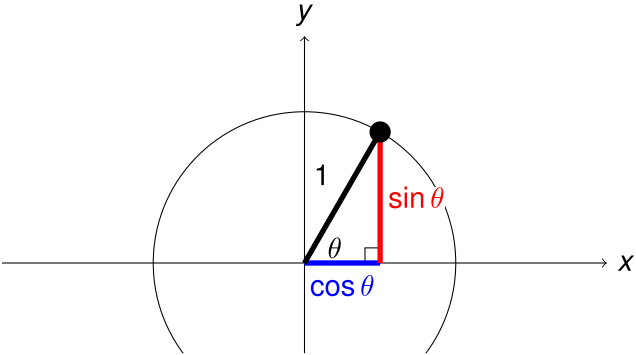
Preliminaries:

- Pythagorean Theorem
- Trig Definition of *sin* and *cos*

Objectives:

- Given $\sin\theta$, find $\cos\theta$
- Given $\cos\theta$, find $\sin\theta$

Pythagorean Theorem



Pythagorean Theorem

$$\cos^2\theta + \sin^2\theta = 1$$

Example 1

If $\cos\theta = \frac{2}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$,

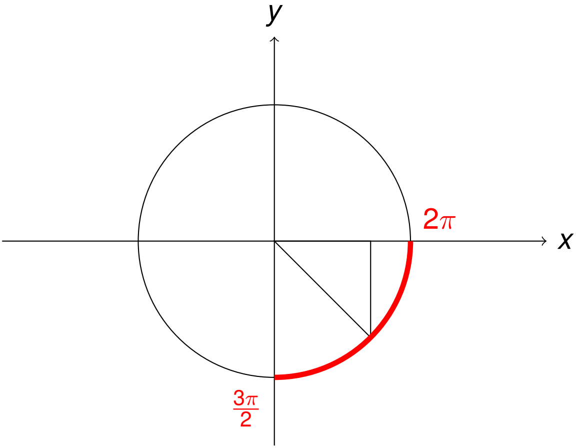
find $\sin\theta$.

Solution:

$$\left(\frac{2}{5}\right)^2 + \sin^2\theta = 1$$

$$\Rightarrow \sin\theta = \pm\frac{\sqrt{21}}{5}$$

Since θ is in quadrant IV, $\sin\theta = -\frac{\sqrt{21}}{5}$.



Further Examples

If $\sin\theta = \frac{3}{7}$ and $\frac{\pi}{2} < \theta < \pi$,

find $\cos\theta$.

If $\cos\theta = -\frac{2}{3}$ and $\frac{\pi}{2} < \theta < \pi$,

find $\sin\theta$.

Further Examples

If $\sin\theta = \frac{3}{7}$ and $\frac{\pi}{2} < \theta < \pi$,

find $\cos\theta$.

Solution:

$$\left(\frac{3}{7}\right)^2 + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{9}{49} = \frac{40}{49}$$

$$\cos\theta = \pm\frac{\sqrt{40}}{7} = \pm\frac{2\sqrt{10}}{7}$$

$$\Rightarrow \cos\theta = -\frac{2\sqrt{10}}{7} \text{ since } \theta \text{ is in quadrant II.}$$

Further Examples

If $\cos\theta = -\frac{2}{3}$ and $\frac{\pi}{2} < \theta < \pi$,

find $\sin\theta$.

Solution:

$$\sin^2\theta + \left(-\frac{2}{3}\right)^2 = 1$$

$$\sin^2\theta = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\sin\theta = \pm\frac{\sqrt{5}}{3}$$

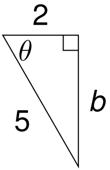
$$\Rightarrow \sin\theta = \frac{\sqrt{5}}{3} \text{ since } \theta \text{ is in quadrant II.}$$

Geometric Technique

If $\cos\theta = \frac{2}{5}$ and θ is in quadrant IV,

find $\sin\theta$.

Solution:



$$2^2 + b^2 = 5^2$$
$$\Rightarrow b^2 = 25 - 4 = 21$$
$$\Rightarrow b = \pm\sqrt{21}$$

Therefore $\sin\theta = -\dfrac{\sqrt{21}}{5}$ since θ is in quadrant IV.

Recap

- Pythagorean Theorem

$$\sin^2\theta + \cos^2\theta = 1$$

- Information about the quadrant will determine whether answer is positive or negative.

- Can use geometric definition of sine and cosine.