Properties of Trig Functions
Preliminaries and Objectives

Preliminaries:
- Values of $\sin$ and $\cos$ for special angles between $0^\circ$ and $360^\circ$ (between 0 and $2\pi$ radians).

Objectives:
- Find values for $\sin$ and $\cos$ for angles larger than $360^\circ$ (larger than $2\pi$).
- Find values for $\sin$ and $\cos$ for negative angles.
Sine and Cosine Values Repeat every $360^\circ$

$$\sin 390^\circ = \sin(360^\circ + 30^\circ) = \sin(30^\circ) = \frac{1}{2}$$
From the original angle, repeatedly subtract 360° until arriving at an angle between 0° and 360°.

Example 1: \[ \sin 390° = \sin(390° - 360°) = \sin 30° = \frac{1}{2} \]

Example 2:
\[
\begin{align*}
\cos 1230° &= \cos(1230° - 360°) = \cos 870° \\
&= \cos(870° - 360°) = \cos 510° \\
&= \cos(510° - 360°) = \cos 150° \\
&= -\frac{\sqrt{3}}{2}
\end{align*}
\]
Sine and Cosine Values Repeat every $360^\circ$

$$\cos \frac{27\pi}{4} = \cos(\frac{24\pi}{4} + \frac{3\pi}{4}) = \cos(3 \cdot 2\pi + \frac{3\pi}{4}) = \cos(\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}$$
Symmetries of the Unit Circle

\[ \sin(-\theta) = -\sin(\theta) \iff \text{TRUE} \]

\[ \cos(-\theta) = -\cos(\theta) \iff \text{FALSE} \]
Symmetries of the Unit Circle

\[
\sin(-\theta) = -\sin(\theta)
\]

\[
\cos(-\theta) = +\cos(\theta)
\]
Symmetries of the Unit Circle

\[ \sin \left( -\frac{\pi}{4} \right) = -\frac{\sqrt{2}}{2} \]

\[ \cos \left( -\frac{\pi}{4} \right) = \frac{\sqrt{2}}{2} \]
Examples

Find \( \cos \left( -\frac{19\pi}{6} \right) = \cos \left( -\frac{7\pi}{6} \right) \cos \left( \frac{5\pi}{6} \right) = -\frac{\sqrt{3}}{2} \)

Find \( \sin \left( -\frac{22\pi}{3} \right) = \sin \left( \frac{2\pi}{3} \right) = \frac{\sqrt{3}}{2} \)
Recap

• To find $\sin$ and $\cos$ values of angles greater than $360^\circ (2\pi)$, subtract full circles to reach an angle on the unit circle.

• To find $\sin$ and $\cos$ values of negative angles, add full circles to reach an angle on the unit circle.