

Finding Values of \tan , \cot , \sec , \csc



Preliminaries and Objectives

Preliminaries:

- Given the value of $\sin \theta$, find the value of $\cos \theta$
- Given the value of $\cos \theta$, find the value of $\sin \theta$

Objectives:

- Define \tan , \cot , \sec , \csc
- Given the value of any of the six trig functions, find the values of the other trig functions.

Definitions

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

Example 1

If $\sin \theta = -\frac{3}{5}$ and $\cos \theta = \frac{4}{5}$, find the values of the other four trig functions.

$$\tan \theta = \frac{-3/5}{4/5} = -\frac{3}{4}$$

$$\cot \theta = -\frac{4}{3}, \quad \sec \theta = \frac{5}{4}, \quad \csc \theta = -\frac{5}{3}$$

Example 2

If $\sin \theta = -\frac{3}{5}$ and $\cos \theta > 0$, find the values of the other five trig functions.

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\left(-\frac{3}{5}\right)^2 + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25}$$

$$\cos \theta = \pm \frac{4}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = \frac{-3/5}{4/5} = -\frac{3}{4}$$

$$\cot \theta = -\frac{4}{3}$$

$$\sec \theta = \frac{5}{4}$$

$$\csc \theta = -\frac{5}{3}$$

Starting with *sec*, or *csc*

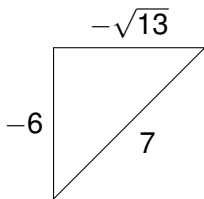
Given $\sec \theta$, first find $\cos \theta$.

Given $\csc \theta$, first find $\sin \theta$.

Example 3

Given $\csc \theta = -\frac{7}{6}$ and $\pi < \theta < \frac{3\pi}{2}$, find the values of the other five trig functions.

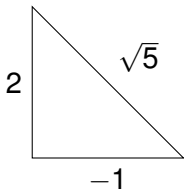
- $\sin \theta = -\frac{6}{7}$
- $\cos \theta = \pm \sqrt{1 - \left(-\frac{6}{7}\right)^2} = -\frac{\sqrt{13}}{7}$
(negative since θ is in quadrant III)
- $\tan \theta = \frac{-6/7}{-\sqrt{13}/7} = \frac{-6}{-\sqrt{13}} = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$
- $\cot \theta = \frac{\sqrt{13}}{6}$
- $\sec \theta = \frac{-7}{\sqrt{13}} = -\frac{7\sqrt{13}}{13}$



Example 4

Given $\tan \theta = -2$ and $\sin \theta > 0$, find the values of the other five trig functions.

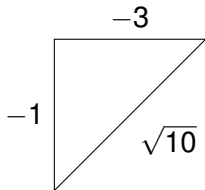
- Since $\sin \theta > 0$ and $\tan \theta < 0$, then $\cos \theta < 0$ and θ is in quadrant II.
- $\sin \theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$
- $\cos \theta = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$
- $\cot \theta = -\frac{1}{2}$
- $\sec \theta = -\sqrt{5}$
- $\csc \theta = \frac{\sqrt{5}}{2}$



Example 5

Given $\cot \theta = 3$ and $\sin \theta < 0$, find the values of the other five trig functions.

- $\tan \theta = \frac{1}{3}$
- Since $\cot \theta > 0$ and $\sin \theta < 0$, then $\cos \theta < 0$ and θ is in quadrant III.
- $\sin \theta = \frac{-1}{\sqrt{10}} = \frac{-\sqrt{10}}{10}$
- $\cos \theta = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$
- $\sec \theta = \frac{-\sqrt{10}}{3}$
- $\csc \theta = -\sqrt{10}$



Recap

- Given $\csc \theta$, take the reciprocal to find $\sin \theta$, then proceed as usual.
- Given $\sec \theta$, take the reciprocal to find $\cos \theta$, then proceed as usual.
- Given $\tan \theta$, draw a right triangle and use the Pythagorean Theorem and the geometric definitions to find $\sin \theta$ and $\cos \theta$.
- Given $\cot \theta$, take the reciprocal to find $\tan \theta$, then follow the tangent procedure.