

## Pythagorean Theorem - Trig Version



## Preliminaries and Objectives

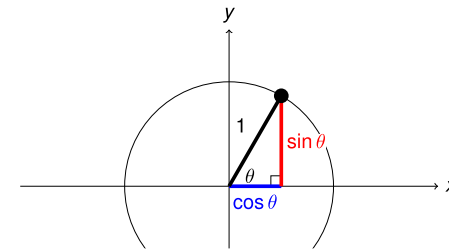
Preliminaries:

- Pythagorean Theorem
- Trig Definition of  $\sin$  and  $\cos$

Objectives:

- Given  $\sin \theta$ , find  $\cos \theta$
- Given  $\cos \theta$ , find  $\sin \theta$

## Pythagorean Theorem



### Pythagorean Theorem

$$\cos^2 \theta + \sin^2 \theta = 1$$

### Example 1

If  $\cos \theta = \frac{2}{5}$  and  $\frac{3\pi}{2} < \theta < 2\pi$ ,

find  $\sin \theta$ .

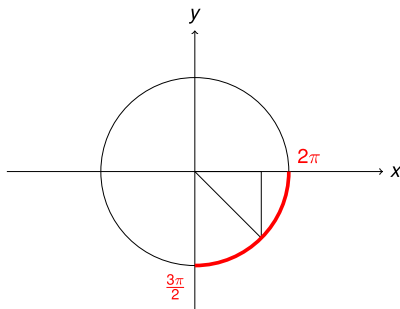
Solution:

$$\left(\frac{2}{5}\right)^2 + \sin^2 \theta = 1$$

$$\Rightarrow \sin \theta = \pm \frac{\sqrt{21}}{5}$$

Since  $\theta$  is in quadrant

$$\text{IV, } \sin \theta = -\frac{\sqrt{21}}{5}.$$



### Further Examples

If  $\sin \theta = \frac{3}{7}$  and  $\frac{\pi}{2} < \theta < \pi$ ,

find  $\cos \theta$ .

If  $\cos \theta = -\frac{2}{3}$  and  $\frac{\pi}{2} < \theta < \pi$ ,

find  $\sin \theta$ .

### Further Examples

If  $\sin \theta = \frac{3}{7}$  and  $\frac{\pi}{2} < \theta < \pi$ ,

find  $\cos \theta$ .

Solution:

$$\left(\frac{3}{7}\right)^2 + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - \frac{9}{49} = \frac{40}{49}$$

$$\cos \theta = \pm \frac{\sqrt{40}}{7} = \pm \frac{2\sqrt{10}}{7}$$

$$\Rightarrow \cos \theta = -\frac{2\sqrt{10}}{7} \text{ since } \theta \text{ is in quadrant II.}$$

### Further Examples

If  $\cos \theta = -\frac{2}{3}$  and  $\frac{\pi}{2} < \theta < \pi$ ,

find  $\sin \theta$ .

Solution:

$$\sin^2 \theta + \left(-\frac{2}{3}\right)^2 = 1$$

$$\sin^2 \theta = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\sin \theta = \pm \frac{\sqrt{5}}{3}$$

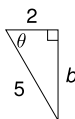
$$\Rightarrow \sin \theta = \frac{\sqrt{5}}{3} \text{ since } \theta \text{ is in quadrant II.}$$

### Geometric Technique

If  $\cos \theta = \frac{2}{5}$  and  $\theta$  is in quadrant IV,

find  $\sin \theta$ .

Solution:



$$\begin{aligned} 2^2 + b^2 &= 5^2 \\ \Rightarrow b^2 &= 25 - 4 = 21 \\ \Rightarrow b &= \pm\sqrt{21} \end{aligned}$$

$$\text{Therefore } \sin \theta = -\frac{\sqrt{21}}{5} \text{ since } \theta \text{ is in quadrant IV.}$$

### Recap

- Pythagorean Theorem

$$\sin^2 \theta + \cos^2 \theta = 1$$

- Information about the quadrant will determine whether answer is positive or negative.

- Can use geometric definition of sine and cosine.