**Pythagorean Theorem - Trig Version**

**Preliminaries and Objectives**

Preliminaries:
- Pythagorean Theorem
- Trig Definition of $\sin$ and $\cos$

Objectives:
- Given $\sin \theta$, find $\cos \theta$
- Given $\cos \theta$, find $\sin \theta$

**Pythagorean Theorem**

$$\cos^2 \theta + \sin^2 \theta = 1$$

**Example 1**

If $\cos \theta = \frac{2}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find $\sin \theta$.

Solution:

$$\left(\frac{2}{5}\right)^2 + \sin^2 \theta = 1$$

$$\Rightarrow \sin \theta = \pm \frac{\sqrt{21}}{5}$$

Since $\theta$ is in quadrant IV, $\sin \theta = -\frac{\sqrt{21}}{5}$. 

**Diagram**
Further Examples

If \( \sin \theta = \frac{3}{7} \) and \( \frac{\pi}{2} < \theta < \pi \), find \( \cos \theta \).

If \( \cos \theta = -\frac{2}{3} \) and \( \frac{\pi}{2} < \theta < \pi \), find \( \sin \theta \).

Further Examples

If \( \sin \theta = \frac{3}{7} \) and \( \frac{\pi}{2} < \theta < \pi \), find \( \cos \theta \).

If \( \cos \theta = -\frac{2}{3} \) and \( \frac{\pi}{2} < \theta < \pi \), find \( \sin \theta \).

Solution:

\[
\left( \frac{3}{7} \right)^2 + \cos^2 \theta = 1
\]

\[
\cos^2 \theta = 1 - \frac{9}{49} = \frac{40}{49}
\]

\[
\cos \theta = \pm \frac{\sqrt{40}}{7} = \pm \frac{2\sqrt{10}}{7}
\]

\( \Rightarrow \cos \theta = -\frac{2\sqrt{10}}{7} \) since \( \theta \) is in quadrant II.

Geometric Technique

If \( \cos \theta = \frac{2}{5} \) and \( \theta \) is in quadrant IV, find \( \sin \theta \).

Solution:

\[
2^2 + b^2 = 5^2
\]

\( \Rightarrow b^2 = 25 - 4 = 21 \)

\( \Rightarrow b = \pm \sqrt{21} \)

Therefore \( \sin \theta = -\frac{\sqrt{21}}{5} \) since \( \theta \) is in quadrant IV.
Recap

- Pythagorean Theorem

\[ \sin^2 \theta + \cos^2 \theta = 1 \]

- Information about the quadrant will determine whether the answer is positive or negative.

- Can use geometric definition of sine and cosine.