

Properties of Trig Functions



Preliminaries and Objectives

Preliminaries:

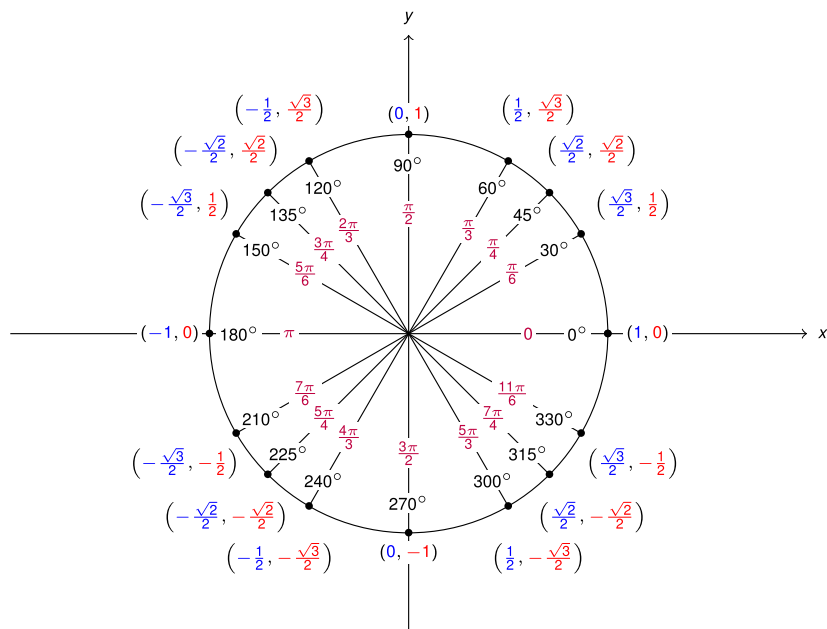
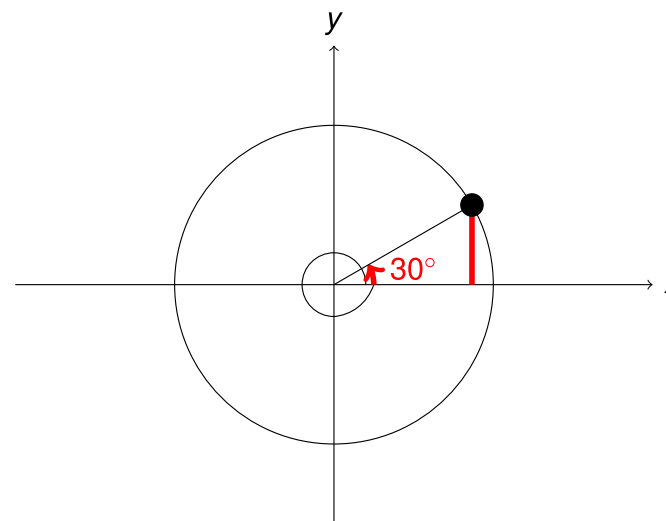
- Values of *sin* and *cos* for special angles between 0° and 360° (between 0 and 2π radians).

Objectives:

- Find values for *sin* and *cos* for angles larger than 360° (larger than 2π).
- Find values for *sin* and *cos* for negative angles.

Sine and Cosine Values Repeat every 360°

$$\sin 390^\circ = \sin(360^\circ + 30^\circ) = \sin(30^\circ) = \frac{1}{2}$$



Solving Process for $\theta > 360^\circ$

From the original angle, repeatedly subtract 360° until arriving at an angle between 0° and 360° .

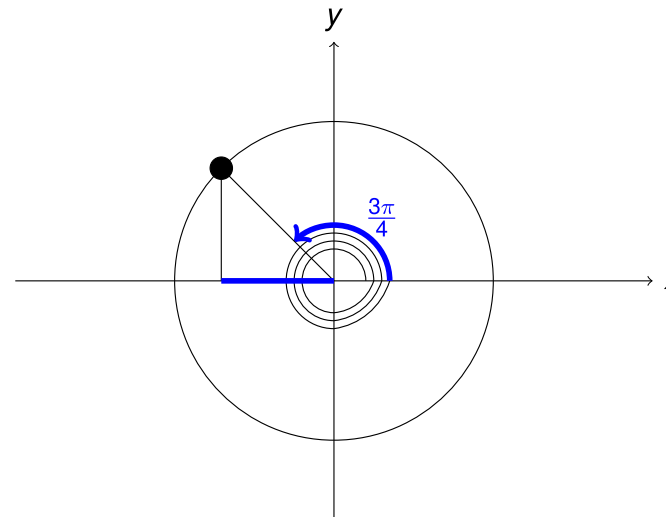
Example 1: $\sin 390^\circ = \sin(390^\circ - 360^\circ) = \sin 30^\circ = \frac{1}{2}$

Example 2:

$$\begin{aligned}\cos 1230^\circ &= \cos(1230^\circ - 360^\circ) = \cos 870^\circ \\ &= \cos(870^\circ - 360^\circ) = \cos 510^\circ \\ &= \cos(510^\circ - 360^\circ) = \cos 150^\circ \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

Sine and Cosine Values Repeat every 360°

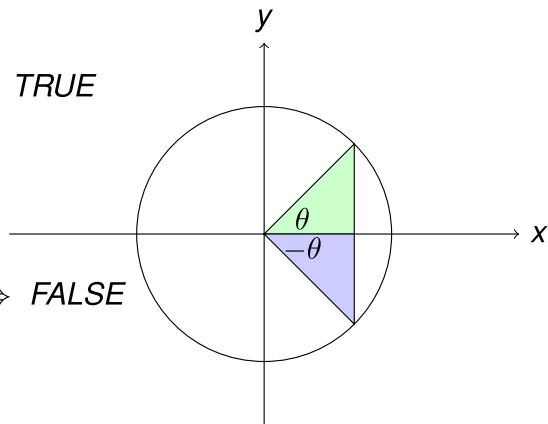
$$\cos \frac{27\pi}{4} = \cos\left(\frac{24\pi}{4} + \frac{3\pi}{4}\right) = \cos(3 \cdot 2\pi + \frac{3\pi}{4}) = \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$



Symmetries of the Unit Circle

$$\sin(-\theta) = -\sin(\theta) \iff \text{TRUE}$$

$$\cos(-\theta) = -\cos(\theta) \iff \text{FALSE}$$



Symmetries of the Unit Circle

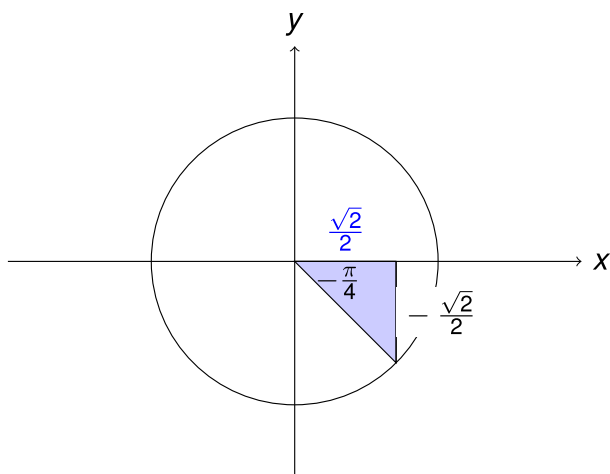
$$\sin(-\theta) = -\sin(\theta)$$

$$\cos(-\theta) = +\cos(\theta)$$

Symmetries of the Unit Circle

$$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$



Examples

$$\text{Find } \cos\left(-\frac{19\pi}{6}\right) = \cos\left(-\frac{7\pi}{6}\right) \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\text{Find } \sin\left(-\frac{22\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Recap

- To find *sin* and *cos* values of angles greater than $360^\circ(2\pi)$, subtract full circles to reach an angle on the unit circle.
- To find *sin* and *cos* values of negative angles, add full circles to reach an angle on the unit circle.