Preliminaries and Objectives

Preliminaries:
- Given the value of $\sin \theta$, find the value of $\cos \theta$
- Given the value of $\cos \theta$, find the value of $\sin \theta$

Objectives:
- Define $\tan$, $\cot$, $\sec$, $\csc$
- Given the value of any of the six trig functions, find the values of the other trig functions.

Definitions

\[
\tan \theta = \frac{\sin \theta}{\cos \theta}
\]
\[
\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}
\]
\[
csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta}
\]

Example 1

If $\sin \theta = -\frac{3}{5}$ and $\cos \theta = \frac{4}{5}$, find the values of the other four trig functions.

\[
\tan \theta = \frac{-3/5}{4/5} = -\frac{3}{4}
\]
\[
\cot \theta = -\frac{4}{3}, \quad \sec \theta = \frac{5}{4}, \quad \csc \theta = -\frac{5}{3}
\]
Example 2

If $\sin \theta = -\frac{3}{5}$ and $\cos \theta > 0$, find the values of the other five trig functions.

\[
\sin^2 \theta + \cos^2 \theta = 1
\]
\[
\left(-\frac{3}{5}\right)^2 + \cos^2 \theta = 1
\]
\[
\cos^2 \theta = 1 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25}
\]
\[
\cos \theta = \frac{4}{5}
\]
\[
\cos \theta = \pm \frac{4}{5}
\]
\[
\tan \theta = \frac{-3}{\sqrt{5}} = -\frac{3}{4}
\]
\[
\cot \theta = -\frac{4}{3}
\]
\[
\sec \theta = \frac{5}{4}
\]
\[
\csc \theta = -\frac{5}{3}
\]

Starting with $\sec$, or $\csc$

Given $\sec \theta$, first find $\cos \theta$.

Given $\csc \theta$, first find $\sin \theta$.

Example 3

Given $\csc \theta = -\frac{7}{6}$ and $\pi < \theta < \frac{3\pi}{2}$, find the values of the other five trig functions.

- $\sin \theta = -\frac{6}{7}$
- $\cos \theta = \pm \sqrt{1 - \left(-\frac{6}{7}\right)^2} = -\frac{\sqrt{13}}{7}$
  (negative since $\theta$ is in quadrant III)
- $\tan \theta = \frac{-6/7}{-\sqrt{13}/7} = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$
- $\cot \theta = \frac{\sqrt{13}}{6}$
- $\sec \theta = -\frac{7}{\sqrt{13}} = -\frac{7\sqrt{13}}{13}$

Example 4

Given $\tan \theta = -2$ and $\sin \theta > 0$, find the values of the other five trig functions.

- Since $\sin \theta > 0$ and $\tan \theta < 0$, then $\cos \theta < 0$ and $\theta$ is in quadrant II.
- $\sin \theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$
- $\cos \theta = \frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$
- $\cot \theta = -\frac{1}{2}$
- $\sec \theta = -\frac{\sqrt{5}}{2}$
- $\csc \theta = \frac{\sqrt{5}}{2}$
Example 5

Given \( \cot \theta = 3 \) and \( \sin \theta < 0 \), find the values of the other five trig functions.

- \( \tan \theta = \frac{1}{3} \)
- Since \( \cot \theta > 0 \) and \( \sin \theta < 0 \), then \( \cos \theta < 0 \) and \( \theta \) is in quadrant III.
- \( \sin \theta = \frac{-1}{\sqrt{10}} = \frac{-\sqrt{10}}{10} \)
- \( \cos \theta = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10} \)
- \( \sec \theta = -\frac{\sqrt{10}}{3} \)
- \( \csc \theta = -\sqrt{10} \)

Recap

- Given \( \csc \theta \), take the reciprocal to find \( \sin \theta \), then proceed as usual.
- Given \( \sec \theta \), take the reciprocal to find \( \cos \theta \), then proceed as usual.
- Given \( \tan \theta \), draw a right triangle and use the Pythagorean Theorem and the geometric definitions to find \( \sin \theta \) and \( \cos \theta \).
- Given \( \cot \theta \), take the reciprocal to find \( \tan \theta \), then follow the tangent procedure.