Finding Values of *tan*, *cot*, *sec*, *csc*

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Preliminaries and Objectives

Preliminaries:

- Given the value of $\sin\theta$, find the value of $\cos\theta$
- Given the value of $\cos\theta$, find the value of $\sin\theta$

Objectives:

- Define *tan*, *cot*, *sec*, *csc*
- Given the value of any of the six trig functions, find the values of the other trig functions.

Definitions

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

Example 1

If $\sin\theta = -\frac{3}{5}$ and $\cos\theta = \frac{4}{5}$, find the values of the other four trig functions.

$$\tan\theta = \frac{-3/5}{4/5} = -\frac{3}{4}$$

$$\cot\theta = -\frac{4}{3}, \qquad \sec\theta = \frac{5}{4}, \qquad \csc\theta = -\frac{5}{3}$$

Example 2

If $\sin\theta = -\frac{3}{5}$ and $\cos\theta > 0$, find the values of the other five trig functions.

$$\sin^2\theta + \cos^2\theta = 1$$
$$\left(-\frac{3}{5}\right)^2 + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25}$$
$$\cos\theta = \pm\frac{4}{5}$$
$$\cos\theta = \frac{4}{5}$$

$$\tan\theta = \frac{-3/5}{4/5} = -\frac{3}{4}$$
$$\cot\theta = -\frac{4}{3}$$
$$\sec\theta = \frac{5}{4}$$
$$\csc\theta = -\frac{5}{3}$$

Starting with *sec*, or *csc*

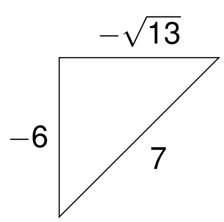
Given $\sec\theta$, first find $\cos\theta$.

Given $\csc\theta$, first find $\sin\theta$.

Example 3

Given $\csc\theta = -\frac{7}{6}$ and $\pi < \theta < \frac{3\pi}{2}$, find the values of the other five trig functions.

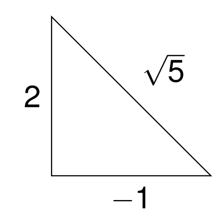
- $\sin\theta = -\frac{6}{7}$
- $\cos\theta = \pm\sqrt{1 - \left(-\frac{6}{7}\right)^2} = -\frac{\sqrt{13}}{7}$ (negative since θ is in quadrant III)
- $\tan\theta = \frac{-6/7}{-\sqrt{13}/7} = \frac{-6}{-\sqrt{13}} = \frac{6}{\sqrt{13}} = \frac{6\sqrt{13}}{13}$
- $\cot\theta = \frac{\sqrt{13}}{6}$
- $\sec\theta = \frac{-7}{\sqrt{13}} = \frac{-7\sqrt{13}}{13}$



Example 4

Given $\tan\theta = -2$ and $\sin\theta > 0$, find the values of the other five trig functions.

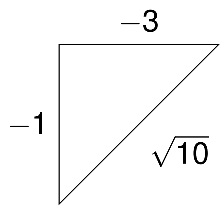
- Since $\sin\theta > 0$ and $\tan\theta < 0$, then $\cos\theta < 0$ and θ is in quadrant II.
- $\sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$
- $\cos\theta = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$
- $\cot\theta = -\frac{1}{2}$
- $\sec\theta = -\sqrt{5}$
- $\csc\theta = \frac{\sqrt{5}}{2}$



Example 5

Given $\cot\theta = 3$ and $\sin\theta < 0$, find the values of the other five trig functions.

- $\tan\theta = \frac{1}{3}$
- Since $\cot\theta > 0$ and $\sin\theta < 0$, then $\cos\theta < 0$ and θ is in quadrant III.
- $\sin\theta = \frac{-1}{\sqrt{10}} = \frac{-\sqrt{10}}{10}$
- $\cos\theta = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$
- $\sec\theta = \frac{-\sqrt{10}}{3}$
- $\csc\theta = -\sqrt{10}$



Recap

- Given $\csc\theta$, take the reciprocal to find $\sin\theta$, then proceed as usual.
- Given $\sec\theta$, take the reciprocal to find $\cos\theta$, then proceed as usual.
- Given $\tan\theta$, draw a right triangle and use the Pythogorean Theorem and the geometric definitions to find $\sin\theta$ and $\cos\theta$.
- Given $\cot\theta$, take the reciprocal to find $\tan\theta$, then follow the tangent procedure.