1. The Radian Measure of an Angle

2. You should be familiar with solving ratio and proportion problems and should be familiar with the unit circle, measured in degrees, including angles measuring over $180°$ and negative angles. In this lesson, we introduce a new angle measure called radian measure, and use it to convert from degrees to radians and from radians to degrees.

3. We can find sine and cosine values of certain angles measured in degrees. These values were derived from points located on a circle with radius 1, called the unit circle. We will use the unit circle to derive an alternate way to measure angles, called radian measure. Our initial definition of sine and cosine came from an association between angles and points on the unit circle. This time, we will focus on the point on the unit circle, and how far it is around the circle from the starting point of $(1, 0)$, represented here by the red arc. If the radius is 1, the circumference of the circle is 2π, so a full circle is 2π. Half a circle therefore is π, so the point at the left edge of the circle is a distance of π from the starting point. The point at the top of the circle is a quarter of a circle away from the starting point, and one-quarter of a circle is $\frac{\pi}{2}$. The point three-quarters of the way around the circle will have a red arc whose distance is $\frac{3\pi}{2}$.

4. Here are the four main points, along with their distances from the starting point at $(1, 0)$, counterclockwise along the circular path. We will now associate these distances with the angles. The $90°$ angle, which corresponded to the point $(0, 1)$ at the top of the circle will now be called $\frac{\pi}{2}$ radians. The $180°$ angle, which corresponded to the point $(-1, 0)$ at the left edge of the circle will now be called π radians. The full circle of $360°$ is 2π radians and the angle $270°$ is $\frac{3\pi}{2}$ radians.

5. We now wish to convert from radians to degrees and vice versa. The key conversion factor is derived from the fact that a full circle is $360°$ and also 2π radians. It may be simpler to use the half-circle conversion, that π radians is $180°$.

6. (a) In most familiar cases, the radian measure will be a rational number times π, in which case there is an easy way to make the conversion. We know that π radians is $180°$, so we can merely replace the π with $180°$.
   (b) We can then simplify.

7. (a) When converting from degrees to radians, one simple approach is to set up a proportion. We know the standard is that π radians is $180°$, so we will use this as our standard ratio.
   (b) For the other ratio, we wish to find the radian measure of $315°$. Notice that the radian measure appears in the numerator of both ratios. You could also do this problem with the degree measure in both numerators. Either way, we cross-multiply to reach the answer.

8. (a) This same method can be used when the radian measure does not include π. The standard ratio is $180°$ is π radians,
   (b) we now wish to find the degree measure for 4 radians. Again, we set up a proportion, and then cross-multiply and simplify to find the answer.

9. To recap: We define radian measure to agree with the circumference of the circle of radius 1. In general, the length of the arc is the radian measure. The circumference of a circle with radius 1 is 2π, which gives us the standard conversion that half a circle is π radians, and also $180°$.