1. Pythagorean Theorem - Trig Version

2. You should be familiar with the geometric version of the Pythagorean Theorem and the trig definition of sine and cosine. That is, sine and cosine are defined as the coordinates of points on a circle of radius 1, called the unit circle.

   In this lesson, we will find the value of sine, given the value of cosine, and vice versa. Note that we do not need to know the angle; the value of one trig function leads directly to the value of the other.

3. (a) We already know the Pythagorean Theorem for right triangles.
   (b) We now wish to apply it to the right triangles that we see in the unit circle. $\cos\theta$ is the x-coordinate of the black dot, but it is also the length of the bottom side of the triangle. $\sin\theta$ is the y-coordinate of the black dot, but it is also the length of the leg opposite the angle θ.
   (c) Using these lengths, the Pythagorean Theorem is $\cos\theta$ squared plus $\sin\theta$ squared equals 1^2, which is one.
   (d) As a shorthand notation, we typically write the square on the cosine and sine functions, and omit the parentheses. This equation will enable us to find the sine value, when the cosine value is known, and vice versa.

4. (a) For example, find $\sin\theta$ if $\cos\theta = \frac{2}{5}$.
   (b) We begin by using the Pythagorean Theorem to find that the square of $sin\theta$ is $21/25$. We get two possible answers, one positive, one negative.
   (c) In order to find the precise value for $\sin\theta$, we would need to be given information about the location of the angle. Since $\cos\theta$ is positive, the angle must either be in the first quadrant or the fourth quadrant.
   (d) One way that we could ask the question is to specify the quadrant specifically. In this case, if you are given that $\cos\theta$ is a positive number, and the angle is in the fourth quadrant, you would then know that $\sin\theta$ would be a negative number. So you would choose the negative number after using the Pythagorean theorem.
   (e) Another way to ask the question is to specify the size of θ. You could be given that $\cos\theta$ is a positive number, and that θ itself is between $\frac{3\pi}{2}$ and 2π. You would then know that θ is in the fourth quadrant, and $\sin\theta$ should be a negative number.
   (f) Again, you will choose the negative number after applying the Pythagorean theorem.

5. Here are two more examples. You may wish to pause the video here in order to work out these problems.

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8. Note that you could also solve these problems using the geometric definition. Instead of labelling the triangle with the hypotenuse equal to 1, you can label the triangle so that the adjacent side is 2 and the hypotenuse is 5, to make the value $\cos\theta = \frac{2}{5}$. You can then find the missing side by the Pythagorean Theorem, and use the values of opposite over hypotenuse to find $\sin\theta$.

9. To recap: On the unit circle, the Pythagorean Theorem becomes $\sin^2\theta + \cos^2\theta = 1$. Given the value of either sine or cosine, you can find the other from the Pythagorean Theorem, except that you will get two answers, one positive, one negative. Information about the quadrant will allow you to determine whether the correct answer is positive or negative.