1. Properties of Trig Functions

2. You should be familiar with the unit circle and how to find sine and cos values for special angles between $0°$ and $360°$

   In this lesson, we will find sine and cosine values for special angles larger than $360°$ and for negative angles.

3. We have developed a unit circle that will help us find the values of sine and cosine from $0°$ to $360°$ for certain angles. We can get other values by realizing that once we reach $360°$, we start over, and the values of sine and cosine repeat.

4. (a) For example, to arrive at $390°$, we travel a full circle of $360°$ plus an extra $30°$.

   (b) So $\sin 390°$ is the same as $\sin 30°$, since $390°$ and $30°$ correspond to the same point on the unit circle. In this case, the sine value is $\frac{1}{2}$.

5. We can find sine and cosine values for angles bigger than $360°$ as follows: From the original angle, subtract full circles of $360°$, until reaching an angle between $0°$ and $360°$. For example, from $390°$, go back one full circle to $30°$. In example 2, we subtract a full circle from $1230°$, to get $870°$. Another full circle to get $510°$. Yet one more full circle to get $150°$, whose cosine value we can look up on the unit circle.

6. (a) The same is true for radians. To find $\cos \frac{27\pi}{4}$, we realize that 2π is a full circle.

   (b) Using a common denominator, $\frac{8\pi}{4}$ is one full circle, $\frac{16\pi}{4}$ is two full circles, $\frac{24\pi}{4}$ is three full circles, and $\frac{32\pi}{4}$ is four full circles.

   (c) From $\frac{27\pi}{4}$, we can subtract three full circles, which is $\frac{24\pi}{4}$, and arrive at the same location as $\frac{3\pi}{4}$, and look up $\cos \frac{3\pi}{4}$, which is $-\frac{\sqrt{2}}{2}$.

7. (a) We can use the symmetry of the circle, to find the value of sine and cosine for negative angles. Are the following statements true or false? Let's think about what these equations say for angles between $0°$ and $90°$. When the original angle is in the first quadrant, the corresponding negative angle will be in the fourth quadrant. Sine values in the fourth quadrant are negative, while cosine values in the fourth quadrant are positive.

   (b) Comparing sine values, the sine value in the first quadrant is positive, while the sine value in the fourth quadrant is negative, so the first equation is true, $\sin(-\theta)$, the fourth quadrant angle is the opposite of the sine of the first quadrant angle.

   (c) Let's take a close look at the top equation again. The left hand side is the sine of a fourth quadrant angle, which is a negative number since the vertical side of the triangle is below the axis.

   (d) On the right side, $\sin(\theta)$ is positive, since the vertical side of the triangle is above the axis, and then we add the negative sign to make the right hand quantity negative. Both the left hand side and right hand side are negative.

   (e) In the bottom statement, the cosine of both positive θ and $-\theta$ correspond to the side of the triangle which is to the right of the axis. Therefore the left side of the bottom equation is a positive number, since the horizontal edge of the blue triangle points to the positive side of the x-axis. The right side has an extra negative sign in front of $\cos \theta$ so the bottom equation is false.

(f) We should have instead that $\cos(-\theta) = \cos\theta$.

8. These two statements are correct.

9. (a) Here is an example of how we can use our knowledge of first quadrant positive angles to help us find values for negative angles. $\sin(-\frac{\pi}{4})$ is the sine of a fourth quadrant angle, and therefore should be a negative number, specifically $-\frac{\sqrt{2}}{2}$.

   (b) $\cos(-\frac{\pi}{4})$ is the cosine of a fourth quadrant angle, and therefore should be a positive number, specifically positive $\frac{\sqrt{2}}{2}$.

   (c) .

10. (a) We can also add full circles to find values for sine and cosine. Here are two examples. In the first example, we can add a full circle of 2π, which is $\frac{12\pi}{6}$, to arrive at $-\frac{7\pi}{6}$.

    (b) You may count around the unit circle backwards to find this value, or you may wish to add another full so that the angle is positive.

    (c) $\cos(-\frac{19\pi}{6}) = \cos(\frac{5\pi}{6})$, which is $-\frac{\sqrt{3}}{2}$. You may wish to pause the video here to work out the second example.

    (d) By adding $\frac{6\pi}{3}$ four times, we arrive at $\frac{2\pi}{3}$, whose sine value is $\frac{\sqrt{3}}{2}$.

11. To recap: When given an angle larger than 360°, or less than 0°, add or subtract full circles to reach an angle on the unit circle.