The Geometry of Right Triangles

Preliminaries and Objectives

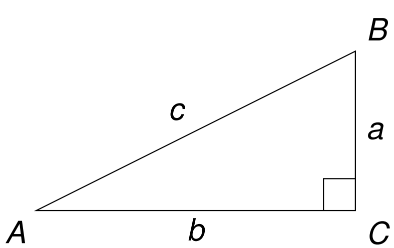
Preliminaries:

- Geometry
- Pythagorean Theorem
- Notation used in Geometry

Objectives:

- Geometric Definition of *sin*, *cos*, and *tan*
- Solve Right Triangles

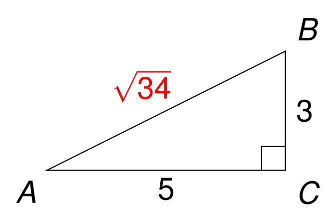
Standard Notation and Definitions



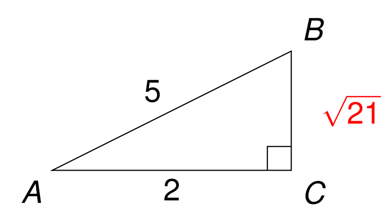
Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Pythagorean Theorem Examples

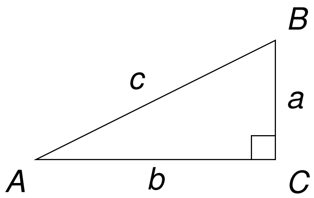


$$5^2 + 3^2 = c^2$$
$$c^2 = 25 + 9 = 34$$
$$c = \sqrt{34} \approx 5.831$$



$$a^2 + 2^2 = 5^2$$
$$a^2 = 25 - 4 = 21$$
$$a = \sqrt{21} \approx 4.583$$

Trig Functions

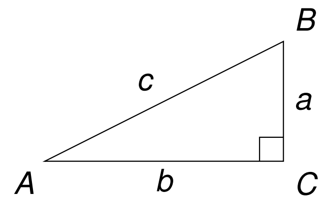


$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

Trig Functions

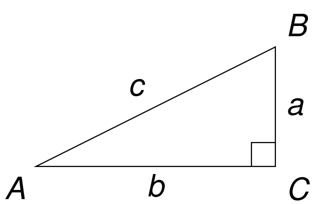


$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a}$$

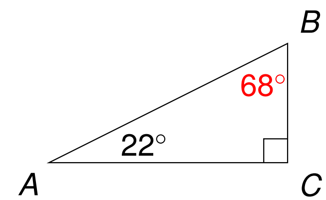
Trig Functions



$$\sin B = \frac{b}{c} = \cos A$$

$$\cos B = \frac{a}{c} = \sin A$$

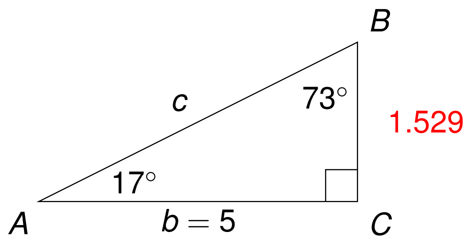
Solving Right Triangles



$$22^\circ + B + 90^\circ = 180^\circ$$

$$B = 180^\circ - 90^\circ - 22^\circ = 68^\circ$$

Solving Right Triangles

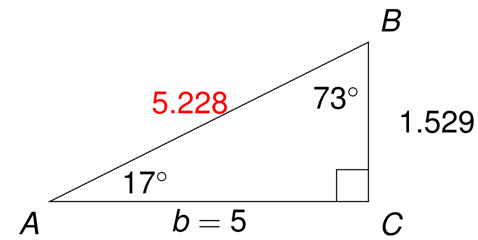


$$B = 90^\circ - 17^\circ = 73^\circ$$

$$\tan A = \frac{a}{b} \Rightarrow \tan 17^\circ = \frac{a}{5}$$

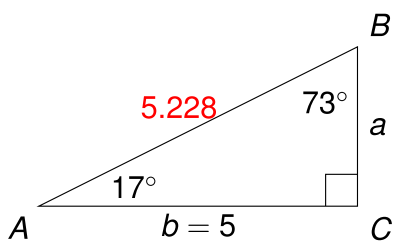
$$\Rightarrow a = 5 \cdot \tan 17^\circ \approx 1.529$$

Solving Right Triangles



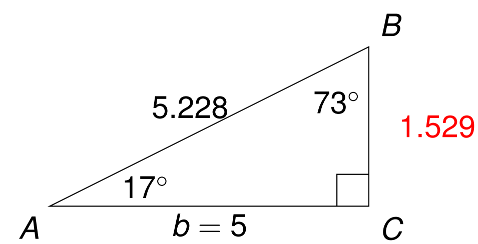
$$(1.529)^2 + 5^2 \approx c^2$$
$$c = \sqrt{25 + 2.338} \approx 5.228$$

Solving Right Triangles



$$\cos A = \frac{b}{c} \Rightarrow \cos 17^\circ = \frac{5}{c}$$
$$\Rightarrow c = \frac{5}{\cos 17^\circ} \approx 5.228$$

Solving Right Triangles

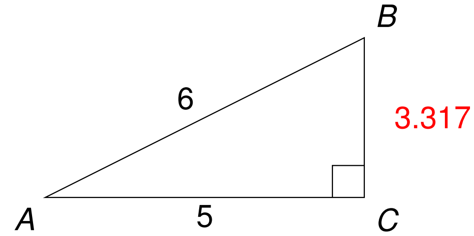


$$\sin A = \frac{a}{c} \Rightarrow \sin 17^\circ \approx \frac{a}{5.228}$$
$$\Rightarrow a = 5.228 \cdot \sin 17^\circ \approx 1.529$$

The Inverse Sine Function

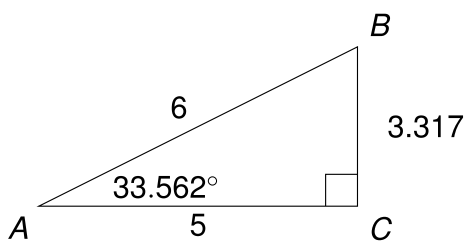
$$\sin^{-1} = \text{arcsin} = \text{inv sin}$$

Solving Right Triangles



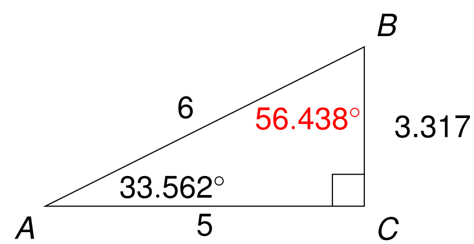
$$a^2 + 5^2 = 6^2$$
$$a = \sqrt{36 - 25} \approx 3.317$$

Solving Right Triangles



$$\sin A = \frac{3.317}{6} \Rightarrow A = \sin^{-1}\left(\frac{3.317}{6}\right) \approx 33.562^\circ$$
$$A = \cos^{-1}\left(\frac{5}{6}\right) \approx 33.562^\circ$$
$$A = \tan^{-1}\left(\frac{3.317}{5}\right) \approx 33.562^\circ$$

Solving Right Triangles



$$\sin A = \frac{3.317}{6} \Rightarrow A = \sin^{-1}\left(\frac{3.317}{6}\right) \approx 33.562^\circ$$
$$B = 90^\circ - 33.562^\circ \approx 56.438^\circ$$