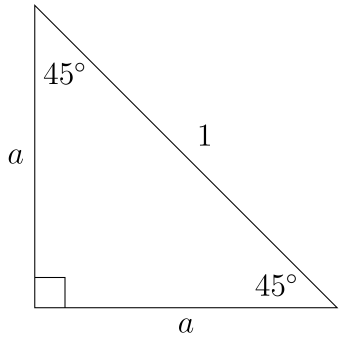
1. For the $45° - 45° - 90°$ triangle, (the isosceles right triangle), there are two legs of length a and the hypotenuse of length 1.



   - Use the Pythagorean Theorem to write an equation relating the lengths of the sides of the triangle.

     For this problem, students should be able to set up the relation $\boxed{a^2 + a^2 = 1}$

     This problem doesn't specifically explain *why* the other two angles in this triangle are each 45°. For students who are working through this problem quickly, ask them to pause and consider why each of the non-right angles are 45°. They may recall some theorems from geometry; base angles of an isosceles triangle are congruent, the sum of angles in a triangle is 180°.

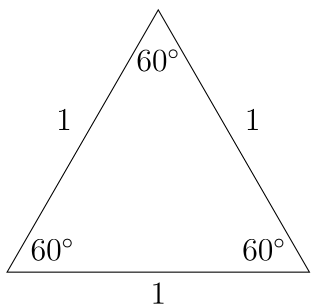
   - Solve the equation for a. (Note: Only the positive answer will make sense.)

     Students may have trouble simplifying the left side to $a^2 + a^2 = 2a^2$. They will be a few students who write $a^2 + a^2 = a^4$. For students who do write this, prompt them to plug in some numbers for a to see if their equality holds true.

     When students take the square root of both sides, ask them why they did or did not include the $\pm\sqrt{}$

     Ultimately, $\boxed{a = \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}}$. This is a good opportunity to ask students about properties of square roots and rationalizing denominators.

2. To find the lengths of the legs of the $30° - 60° - 90°$ triangle, begin with an equilateral triangle, all of whose sides are length 1.



- From the top vertex, draw a line segment perpendicular to the bottom side, cutting the original triangle into to congruent triangles. (Geometry review: The new line segment is called the perpendicular bisector, it is also called the median, it is also called the altitude.)
- Find the lengths of the two halves of the bottom side.
- Find all the angles in the triangles.
- Label the length of the altitude h
- Use the Pythagorean Theorem to write an equation involving h
- Solve the equation for h.

This is an excellent opportunity to get students at the white boards diagramming their thoughts. They will be adding in the altitude, labelling various parts, writing equations and solving. It is sometimes helpful to have the student who has already figured out the answer explain their work to someone else who is writing in the board. Several people can also be at the board in succession, there are several steps.

This problem involves some level of abstraction when it comes to identifying h with a leg and not the $h$ypotenuse. Students should end up with relation $\boxed{h^2 + (\frac{1}{2})^2 = 1}$. Here, there are a few things to note: if some students have $h^2 + (\frac{1}{2})^2$ and others have $(\frac{1}{2})^2 + h^2$, ask them why they got different answers/if it matters; now is also a good time to point out that $(\frac{1}{2})^2 \neq \frac{1^2}{2}$, which some will undoubtedly write; compare the answers whose RHS is 1^2 vs. whose RHS is just 1 (some students may be confused by the dropped exponent).

The final answer should be $\boxed{h = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}}$. Again, this is a good opportunity to talk about rules for simplifying fractions and square roots.

3. Draw the 45-45-90 triangle in as many orientations as possible, keeping the legs either horizontal or vertical. (Hint: You can rotate and reflect the triangle)

   There are $\boxed{4}$ ways. Ask students how they know they have found ALL of the possibilities, both on this problem and the next.

4. Draw the 30-60-90 triangle in as many orientations as possible, keeping the legs either horizontal or vertical.

   There are $\boxed{8}$ ways.

Because the above two problems are not computational, they are often overlooked. Encourage students to think about *why* we're asking these questions. Perhaps they could draw all of the 12 angles above on an xy-plane. The orientations of these angles will eventually lead into the standard angles on the Unit Circle; the side lengths of these triangles will become the coordinates of points on the xy-plane and the sines and cosines of multiples of $30°, 45°$, and $60°$-degree angles.