The purpose of this worksheet is to derive formulas for the areas of SAS and AAS triangles. To prepare for this worksheet, students should be able to apply the techniques of Topic 1.1 (Solving Right Triangles). It may also be beneficial to remind students of the 'Piston Motion' worksheet, in which they also divided non-right triangles into two right triangles. A common mistake on this worksheet will be to think that angle B is 90 degrees.

Assume we know the formula for the area of a triangle

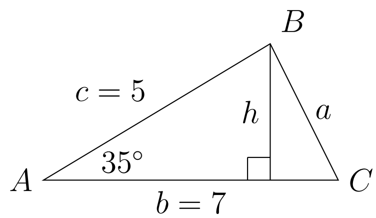
$$Area = \frac{1}{2}(base)(height)$$

1. (SAS) Do as many of the following problems as are necessary for you to develop a process that you can describe in question 2. In each case, find h and the area of the triangle. Note that b is the entire length from A to C, not just the portion that would be the adjacent side to angle A in the right triangle.

   For students who are struggling to start the worksheet, use a piece of paper or your hand to cover the right half of the triangle (or have the student draw the picture on the white board, then erase half of the picture). They can solve the triangle as they would have on Topic 1.1 - Solving Right Triangles.

   Some students may be able to describe the process after just one example. This is a good opportunity to check that everybody understands the process. It is good to ask other students to do further examples to make sure that everyone is familiar with the process before moving on.

   (a) Given $b = 7$, $c = 5$ and $A = 35°$, find h and the area of the triangle.

   (b) Given $b = 12$, $c = 8$ and $A = 52°$, find h and the area of the triangle.

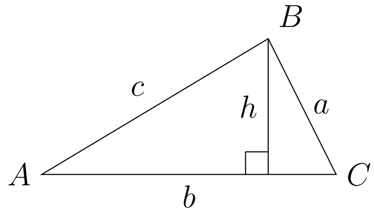
   (c) Given $b = 4$, $c = 11$ and $A = 83°$, find h and the area of the triangle.

   (d) Given $b = 10$, $c = 9$ and $A = 115°$, find h and the area of the triangle.

2. Describe, in words, the steps needed to find the area of a triangle, given A, b, and c. (You may also use mathematical expressions in your description.)

   Typically, student will write down $\sin A$, then solve for h, then plug into the initial area formula. Some will write this in words, others will use mathematical expressions.

3. Using c and A, write a formula for h. Then write a formula for the area of the triangle.

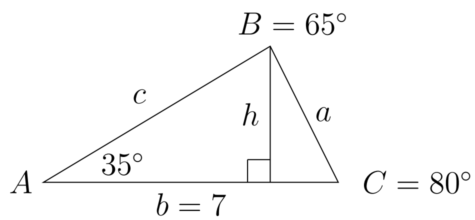


If students are having trouble here, they should be reminded of what they wrote for question 2) and follow their own algorithm. At first, they merely need a formula that involves h, then they can solve for h, then substitute.

4. Repeat using a and C. That is, using a and C, write a formula for h. Then write a formula for the area of the triangle.

5. (AAS = AAAS = ASA) Do as many of the following problems as are necessary for you to develop a process that you can describe in question 6. In each case, find c, then find h, then find the area of the triangle. Note that b is the entire length from A to C, not just the portion that would be the adjacent side to angle A in the right triangle.

   (a) Given $b = 7$, $A = 35°$, $B = 65°$ and $C = 80°$

   - Find c
   - Find h
   - Find the area of the triangle

   (b) Given $b = 12$, $A = 52°$, $B = 67°$ and $C = 61°$

   (c) Given $b = 5$, $A = 85°$, $B = 23°$ and $C = 72°$

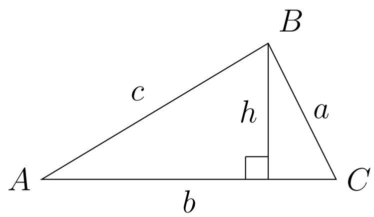
   (d) Given $b = 11$, $A = 115°$, $B = 43°$ and $C = 22°$

Again, this is a chance to make sure all students know the process before moving on.

6. Describe, in words, the steps needed to find the area of a triangle, given b, A, B, and C. (You may also use mathematical expressions in your description.)

Some students will use the Law of Sines to find c, and then realize that they now can use the previous SAS formula. Others will continue a step-by-step process. It is best to let students do what they are comfortable with. The two methods can be compared later.

7. Derive a formula for the area of a triangle, given b, A, B and C, by doing the following



- Find c, as a function of b, C and B
- Find h, as a function of c and A
- Find h, as a function of b, A, B and C
- Find the area of the triangle, as a function of b, A, B and C

You may need to inform students what is meant by 'as a function of'. One key point to emphasize is that we are originally given b, A, B, and C, so it is best if we have a formula that only uses those variables. That way, we can calculate the answer directly from the formula without any intermediary steps.

Ask students what types of triangles they can use this formula for. Will it work for ASA? Will it work for SAS? AAA? AAS?