1. The Geometry of Right Triangles

2. You should be familiar with some basic notions from geometry, like the Pythagorean Theorem, and knowledge of naming conventions is helpful, though in this lesson, all of the ideas will be presented from the beginning. In this lesson, we will give the geometric definition of three functions *sin*, *cos*, and *tan*, and use them to find the three missing parts of a right triangle when three parts are known.

3. (a) Let's begin with points A, B, and C,
   (b) which form a right triangle,
   (c) with the right angle at C. We typically refer to the angles of the triangle, by the point which is at the vertex of the angle, for instance, angle C is the right angle. We typically label the sides of the triangle with a lower case letter corresponding to the angle which is across from it, for instance, the side connecting point A and point C
   (d) is called side b.
   (e) Sides a and b are called the legs, and side c is called the hypotenuse.
   (f) Recall that given the lengths of any two sides of a right triangle, we can find the length of the third side by using the Pythagorean Theorem.

4. (a) Here are two examples. You may wish to pause this video in order to work out the problems yourself.
   (b) .

5. (a) For both angle A and angle B, six trig functions are defined. In this lesson, we will define the three most common trig functions, sine, cosine and tangent.
   (b) The sine of angle A is the length of the leg opposite angle A divided by the length of the hypotenuse. In this case, the sine is a divided by c. Sine is spelled s-i-n-e, but is abbreviated s-i-n.
   (c) The cosine of angle A is the length of the leg adjacent to the angle divided by the length of the hypotenuse. In this case, the cosine is b divided by c. Cosine is abbreviated c-o-s.
   (d) The tangent of angle A is the length of the leg opposite angle A divided by the length of the leg adjacent to angle A. In this case, the tangent is a divided by b. Tangent is abbreviated t-a-n.

6. The sine, cosine and tangent of angle B are defined in the same manner.

7. (a) Notice that the leg opposite angle B is the leg adjacent to angle A, so the sine of B equals the cosine of A.
   (b) Similarly, the leg adjacent to angle B is opposite angle A, so the cosine of B equals the sine of A.

8. (a) Given the measure of angle A in a right triangle, we then know the measure of angle B, since the three angles of a triangle add to $180°$. For example, if angle A is $22°$,
   (b) then angle B will be 68 degrees.

9. (a) Given the length of one leg and the measure of one angle, we can find the missing parts of the triangle. For example, here we have the triangle with the angle A equal to $17°$ and the adjacent leg b having length 5. We can first find angle B since angle A and angle B add to $90°$.

   (b) Angle B will be $73°$. We can then find the lengths of the sides in a variety of ways: We could use the tangent of A to find the length of the missing leg. The tangent of A is the opposite over the adjacent, which in this case, is a over 5.

   (c) We can then solve for a by multiplying the $\tan 17°$ by 5.

   (d) You will need a calculator to get an approximate answer.

10. We could then use the Pythagorean Theorem to find the length of the hypotenuse.

11. Another approach is that we could have used $\cos A$ to find the length of the hypotenuse

12. and then used $\sin A$ to find the missing leg.

13. If you are given the lengths of two of the sides of the triangle, you can find the third side by the Pythagorean Theorem. But how do we find the angles? We will talk in a later lesson about inverse functions, for now, we will use the inverse sine, inverse cosine and inverse tangent keys on a calculator.

    (Supplemental graphic)

    Here, they are written in green next to the corresponding trig function, and are written as the trig function followed by a -1.

14. The inverse sine may also be written arcsin or *inv sin*, and similarly for inverse cosine and inverse tangent. In essence, an inverse function is a reverse lookup.

    (Supplemental graphic)

    Suppose we had a chart that listed angles in the left column, and sine values in the right column. The standard lookup is used when trying to find the sine of an angle by finding the angle in the left column and looking at the answer in the right column. For example, if the angle is 6 degrees, the sine value is .1045. In our case, we will be given the answer, the sine value, and wish to find the angle. For example, if the sine value is .8746, the angle is 61 degrees. On a calculator, the inverse sine button does this, it gives as its output, an angle, when the input is the number which is the sine value.

15. (a) We can find the missing parts of this right triangle first using the Pythagorean Theorem,

    (b) and then finding one angle by the inverse sine.

16. (a) On your calculator, divide 3.317 by 6 to get .55283, then hit the inverse sine key. Make sure your calculator is in degree mode. We could also use the inverse cosine or inverse tangent to calculate the angle.

    (b) We then subtract angle A from $90°$ to find angle B.

17. To recap: When given one acute angle and one side length of a right triangle, find the remaining angle by making the three angles add to $180°$. Then find one of the remaining sides using one of the trig functions. The final side can be found either by using the Pythagorean Theorem or by once again using one of the trig functions. When given two side lengths, one

angle will need to be found using an inverse trig function. The remaining parts can be found as before, using either the fact that the three angles of a triangle add to $180°$, using the Pythagorean Theorem, or using one of the trig functions.