Probability
Activity 1a - Pascal's Triangle Instructor's Guide

The goal of the Instructor's guides is to give instructors an idea of what the goal of the worksheet is, what the answers to some of the questions are, and provide questions to ask students who are speeding through the worksheets or perhaps missing out on certain details.

This activity works best if students form groups of four, and each is assigned one of the four parts below. Students then move around the room so that all of the students working on Problem 1 are working together, etc. After working in these secondary groups, students return to their original group and teach each other what they have learned. As they do so, it is important to try to figure out what the connections are between the four problems. Are they really all the same problem?

The main goal is to discover that the recursion for Pascal's triangle also applies to the other problems. In particular, the number of subsets of size k chosen from a set of size n, called *combinations*, follows the same recursion as Pascal's triangle.

1. Search the internet to find Pascal's Triangle and as much other information as you can find.

   - What is the recursion to get from one row to the next? (Be ready to teach your classmates how to find the next row.)

     This establishes the foundation for the other groups. Make sure the students are good at describing the recursion. It may be good to establish the naming convention $\binom{n}{k}$, the n^{th} row begins $1\ n\ \ldots$, the 1 at the very top is row 0. The k^{th} entry in row n counts from left to right, starting at 0. For example, in the 6th row – 1, 6, 15, 20, 15, 6, 1 – the leading 1 is entry 0, the first entry is 6, the second entry is 15, etc., so that $\binom{n}{k}$ is the k^{th} entry in row n.

   - Do you notice any other patterns in the numbers?

     This is basically a time killer to let the other groups finish, though there are a lot of cool patterns they will find.

2. (a) Write out all sequences of heads and tails on two coin flips. How many have zero heads? one head? two heads?

   Make sure they get both HT and TH. Some students may use the tree diagram. They should figure out a good organization plan.

   (b) Write out all sequences of heads and tails on three coin flips. How many have zero heads? one head? two heads? three heads?

   Get students thinking about how the previous answer helps get this answer.

   (c) Write out all sequences of heads and tails on four coin flips. How many have zero heads? one head? two heads? three heads? four heads?

   Good organization is key here. Students may need to rewrite this to make it look nice. Get students thinking about patterns that they see. How do they know that they have listed all of the possibilities.

   (d) Write out all sequences of heads and tails on five coin flips. How many have zero heads? one head? two heads? three heads? four heads? five heads?

   (e) Are there patterns that you see? Can you describe how to use the information from the previous step to help with the answer for the next step?

3. (a) FOIL $(x+y)^2$. Write out all four terms literally (do not change the order of the letters at first). Then simplify.

   $xx+xy+yx+yy=x^2+2xy+y^2$. It may be good to start with $(a+b)(c+d)$ to get the idea.

   (b) FOIL $(x+y)^3$. Write out all eight terms literally. How does the previous answer help this process? Then simplify.

   (c) FOIL $(x+y)^4$. Write out all 16 terms literally. How does the previous answer help this process? Then simplify.

   (d) FOIL $(x+y)^5$. Write out all 32 terms literally. How does the previous answer help this process? Then simplify.

   In the end, they should be saying the same thing as group 2, using x for H and y for T.

4. (a) Write out all of the sets of one letter from the set $\{A, B\}$. all the sets of two letters.

   A; B (two sets of size 1) + AB (one set of size 2)

   (b) Write out all of the sets of one letter from the set $\{A, B, C\}$. all the sets of two letters, three letters.

   A; B; C (three sets of size 1) + AB; AC; BC (three sets of size 2) + ABC (one set of size 3)

   (c) Write out all of the sets of one letter from the set $\{A, B, C, D\}$. all the sets of two letters, three letters, four letters.

   (d) Write out all of the sets of one letter from the set $\{A, B, C, D, E\}$. all the sets of two letters, three letters, four letters, five letters.

   If we include the empty set, then this list has 32 entries, broken up into components of sizes 1,5,10,10,5,1 respectively, which agrees with Pascal's triangle. Which sequence of Heads and Tails does the subset {A, C, D} correspond to?

After each group has completed their task, the original groups should be reformed. The first thing to do is to have the student who did question 2 to explain to the other group members how the recursion works. Then repeat for questions 3 and 4. Finally, have the question 1 student explain Pascal's triangle.

Then groups should work together to find the connections among the problems. Why are questions 2 and 3 really the same question? How do we translate from coin flips to binomial expansion? Why are the coefficients in the binomial expansion the same as the entries in Pascal's triangle? Why are the numbers of subsets in question 4 the same as the entries in Pascal's triangle? How do we translate from subsets in question 4 to coin flips in question 2?