This activity is designed to reinforce ideas of probability once students are used to working with the *General Counting Principle, Permutations, Combinations,* general definitions from probability include how to decompose problems using *AND/OR/NOT*, and *Binomial Probabilities.*

The problems range from simply using a definition to very difficult. It is important to nut rush students through these problems. Very few students will complete these in a an hour. It is important for students to try different approaches, and when possible, to diagram the list of possibilities. If nothing else, students can get started by listing one possible way something can happen, then another ... Once a partial list is developed, students should try to organize the list, identify components, count components ...

1. A jar contains 15 ping pong balls, 3 are red, 5 are blue, 7 are green. The red balls are numbered 1, 2, 3 to tell them apart. The blue balls are numbered 4, 5, 6, 7, 8. The green balls are numbered 9, 10, 11, 12, 13, 14, 15.

   (a) Draw one ball. What is the probability that the ball is red?

   This is a simple counting problem. $\boxed{\frac{3}{15} = \frac{1}{5}}$. It is often easier in probability to not simplify fractions as the denominator may reveal the technique used to count the total possibilities.

   (b) Draw one ball. What is the probability that it is either red OR blue? Do this problem in a second way.

   Using the OR conjunction $\boxed{\frac{3}{15} + \frac{5}{15} = \frac{8}{15}}$.

   'red or blue' = 'not green' $\boxed{1 - \frac{7}{15} = \frac{8}{15}}$

   (c) Draw two balls. Pick one ball first, then another ball second (without replacing the first ball). In other words, you can't get the same number twice. What is the probability that both balls are blue? Do this problem 3 ways, first by counting combinations, then by using the probability that the first ball is blue AND the second ball is blue, then by counting the possibilities if the order matters.

   $$\frac{C(5,2)}{C(15,2)} = \frac{10}{105}$$

   $$\frac{5}{15} \cdot \frac{4}{14} = \frac{20}{210}$$

   $$\frac{P(5,2)}{P(15,2)} = \frac{20}{210}$$

   It may be good to have students write out all 10 possibilities by the number of the balls.

(d) Draw two balls. Pick one ball first, then another ball second (without replacing the first ball). What is the probability that the first is blue AND the second is red? What is the probability that one ball is red and the other blue? Do this problem using the word 'OR', also do this problem by some counting technique.

First blue and second red: $\dfrac{5}{15} \cdot \dfrac{3}{14} = \dfrac{15}{210}$

First blue and second red OR first red and second blue: $\dfrac{5}{15} \cdot \dfrac{3}{14} + \dfrac{3}{15} \cdot \dfrac{5}{14} = \dfrac{30}{210}$

One red and one blue out of all combinations: $\dfrac{5 \cdot 3}{C(15,2)} = \dfrac{15}{105}$

Again, it may be good to have students write out all of the possibilities by the number of the balls. One thing to be stressed is that the problem can be done by picking the balls in order (permutation), or without regard to order (combinations) as long as both the numerator and denominator are done the same way.

(e) Draw two balls without replacement. What is the probability that at least one ball is green? Do this in as many ways as possible.

First green + (First NOT green AND Second green) $= \dfrac{7}{15} + \dfrac{8}{15} \cdot \dfrac{7}{14} = \dfrac{154}{210}$

First green + Second green - overlap $= \dfrac{7}{15} + \dfrac{7}{15} - \dfrac{7}{15} \cdot \dfrac{6}{14} = \dfrac{154}{210}$

NOT (Both NOT green) = NOT(First NOT green AND Second NOT green) $= 1 - \dfrac{8}{15} \cdot \dfrac{7}{14} = \dfrac{154}{210}$

(f) Draw four balls without replacement. What is the probability that you get two red, one blue, one green?

Total possibilities $= C(15,4) = 1365$, pick the two red in $C(3,2) = 3$ ways, pick the blue in 5 ways, pick the green in 7 ways. $\boxed{\dfrac{105}{1365}}$

2. You roll five six-sided dice, that are five different colors, (blue, red, black, yellow, green) so you can tell them apart. Each dice roll will be listed A-B-C-D-E in that color order, so that, for example, 1-3-4-2-6 means the blue die is 1, red = 3, black = 4, yellow = 2, green = 6. Rolling 3-1-6-4-2 would be a different outcome. (Some terminology in this problem comes from the game of *Yahtzee*

   (a) How many total outcomes are possible?

   $\boxed{6^5 = 7776}$

   (b) What is the probability of rolling 5-5-5-5-5?

   $\boxed{\frac{1}{7776}}$

   (c) What is the probability of getting the numbers 2, 3, 4, 5, 6 in some order?

   This question is asking how many permutations are there. $\boxed{\frac{120}{7776}}$

   (d) What is the probability of getting a large straight (a large straight is five numbers in sequence, so either the numbers 1 through 5 OR the numbers 2 through 6?

   Using the answer from the previous question. $\boxed{\frac{120}{7776} + \frac{120}{7776} = \frac{240}{7776}}$

   (e) What is the probability of rolling 6 on three of the dice (and something other than 6 on the other two).

   This is a binomial counting question.

   $$\boxed{C(5,3)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 = \frac{250}{7776}}$$

   (f) What is the probability of getting a full house? (A full house is three of one value and two of another)

   Pick the three colors which are the three-of-a-kind, pick the value of the three-of-a-kind, pick the value of the pair. $\boxed{\frac{C(5,3)\cdot 6\cdot 5}{7776} = \frac{300}{7776}}$

3. This problem involves Punnett squares, which are a topic in genetics. There is some useful information here

   `https://en.wikipedia.org/wiki/Punnett_square`

   The dominant gene is 'A' and the recessive gene is 'a'. From each parent, one of the two genes (alleles) is chosen randomly, each with probability 1/2.

   Find the probability that an offspring is of type 'AA', 'Aa' or 'aa' if the parents are

   (a) One parent = 'AA', other parent is 'aa' (This first problem is done for you.)

	a	a
A	Aa	Aa
A	Aa	Aa

   Solution:
   P(offspring= 'AA') = 0/4
   P(offspring= 'Aa') = 4/4
   P(offspring= 'aa') = 0/4

   (b) One parent = 'AA', other parent is 'Aa'

	A	a
A	AA	Aa
A	AA	Aa

   Solution:
   P(offspring= 'AA') = 2/4
   P(offspring= 'Aa') = 2/4
   P(offspring= 'aa') = 0/4

   (c) Both parents = 'Aa',

	A	a
A	AA	Aa
a	Aa	aa

   Solution:
   P(offspring= 'AA') = 1/4
   P(offspring= 'Aa') = 2/4
   P(offspring= 'aa') = 1/4

   (d) Generation X - both parents are 'Aa'. The offspring are Generation Y. Two Generation Y offspring are chosen at random and become the parents of Generation Z. What is the probability that a Generation Z offspring is 'AA'? 'Aa'? 'aa'? (Hint: The denominators for the Generation Y offspring are 4, so there are $4 \times 4 = 16$ possible pairs of Generation Y parents. For each of these 16 possibilities, there are four possible offspring, so the denominators of the answers will naturally be 64.)

   $P(AA) = \frac{16}{64}$ $\quad P(Aa) = \frac{32}{64}$ $\quad P(aa) = \frac{16}{64}$