

Probability

Activity 1b - Gen. Count. Prin., Permutations and Combinations

Instructor's Guide

1. (Group 2) - Write out all sequences of heads and tails on five coin flips that contain two heads.

HHTTT, HTHTT, HTTHT, HTTTH, THHTT, THTHT, THTTH, TTHHT, TTHTH, TTTHH

There are 10 total ways. It is best if students have some systematic way of knowing they have the entire list. You may ask 'How do you know you haven't listed something twice?' 'How do you know that you haven't missed anything?'

2. (Group 3) - Write out all of the terms in the expansion of $(x + y)^5$ that simplify to x^2y^3 . These are sequences of x and y that have five letters.

xxyyy, xyxyy, xyyxy, xyyyx, yxxyy, yxyxy, yxyyx, yyxxy, yyxyx, yyyxx

There are 10 total ways. It is best if students have some systematic way of knowing they have the entire list. You may ask 'How do you know you haven't listed something twice?' 'How do you know that you haven't missed anything?'

3. (a) Pair the two lists up so that you can translate between the two lists.

$H \iff x$ $T \iff y$ interchange x for H and y for T .

- (b) In the case $n = 6$, what sequence of heads and tails corresponds to the expansion $xyyxyx$?

$xyyxyx \iff HTTHTH$

- (c) In the case $n = 7$, what expansion corresponds to the coin flip sequence $HHHTHHT$?

$HHHTHHT \iff xxxyxxy$

4. (Group 4) - Write out all of the subsets of size 2, chosen from the set $\{A, B, C, D, E\}$

AB, AC, AD, AE, BC, BD, BE, CD, CE, DE

5. (a) Find a correspondence from the subsets to the coin flips.

Let A correspond to coin flip 1, B correspond to coin flip 2 etc. If A is in the subset, the the first flip is H , if not, the first flip is T , etc.

- (b) Find a correspondence from the expansions of $(x + y)^5$ to the subsets.

Let A correspond to the first letter, etc. If A is in the subset, the the first letter is x , if not, the first letter is y , etc. It is also good to have students describe the correspondence in the other direction.

- (c) In the case $n = 6$, $k = 4$, what sequence of heads and tails corresponds to the subset $\{A, C, E, F\}$?

HTHTHH

- (d) In the case $n = 7$, $k = 5$ what subset corresponds to the coin flip sequence $HHHTHHT$?

A, B, C, E, F

6. Find a correspondence between the subsets of size 2, chosen from the set $\{A, B, C, D, E, F\}$ and the subsets of size 4, also chosen from the set $\{A, B, C, D, E, F\}$.

(Hint: This question is really asking why Pascal's Triangle should be symmetric. Why are the rows of Pascal's Triangle the same, whether read from left to right or from right to left? What is the connection between choosing two things and choosing all but two things?)

$A, B \iff C, D, E, F$
 $A, C \iff B, D, E, F$
 $A, D \iff B, C, E, F$
 $A, E \iff B, C, D, F$
 $A, F \iff B, C, D, E$
 $B, C \iff A, D, E, F$
 etc.

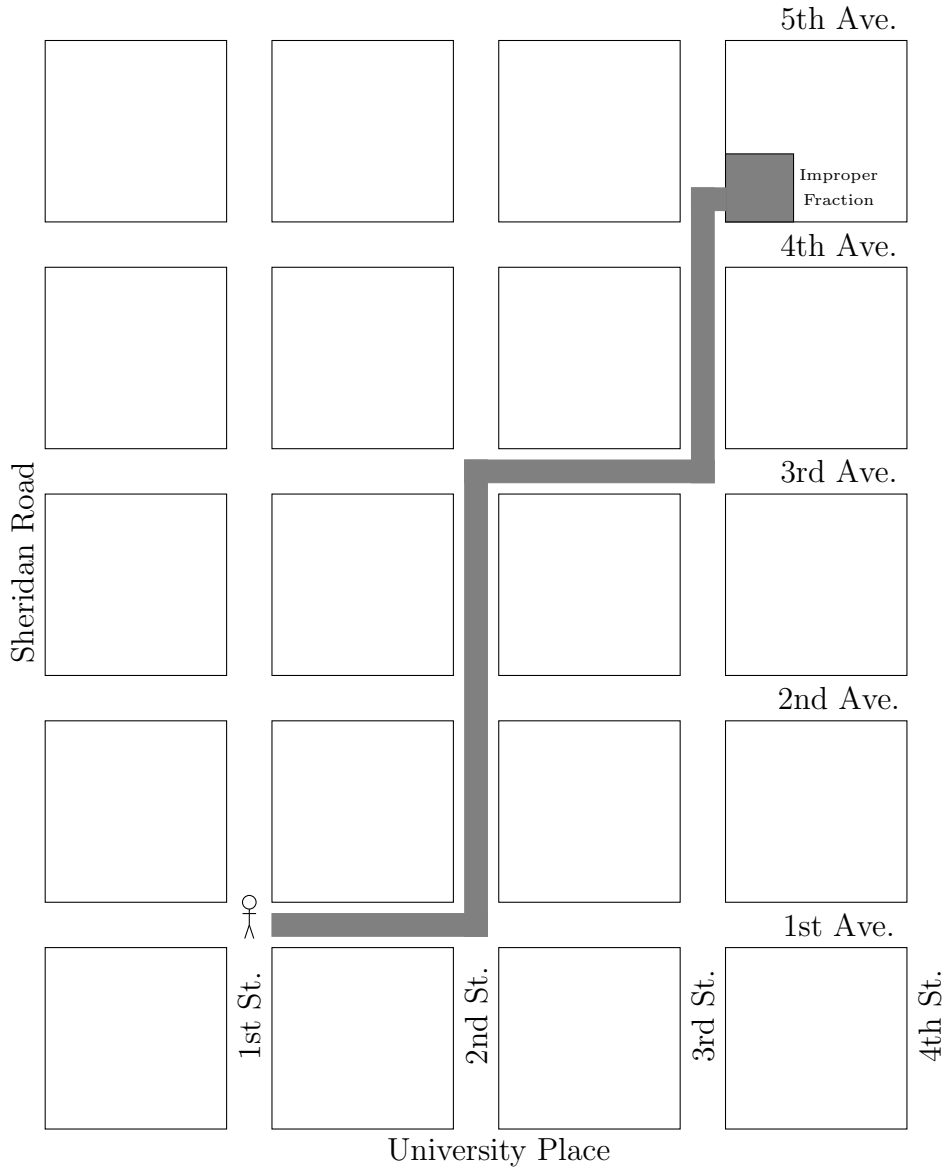
The letters present on the left side are the letters missing from the right side. This shows why Pascal's Triangle is symmetric.

7. (Group 1) - How can you use the 4th row of Pascal's Triangle to get the 5th row of Pascal's Triangle?
8. (a) Describe how to use subsets of size 1 and 2 chosen from the set $\{A, B, C, D\}$ to find the subsets of size 2, chosen from the set $\{A, B, C, D, E\}$
 Keep the subsets of size 2, add E to the subsets of size 1. Students should be encouraged to make the connection to the Pascal's Triangle recursion.
- (b) Describe how to use the expansion of $(x + y)^4$ to find the coefficient on x^2y^3 in the expansion of $(x + y)^5$
 Multiply $4xy^3$ by x and $6x^2y^2$ by y and combine like terms. Students should be encouraged to make the connection to the Pascal's Triangle recursion.
- (c) Describe how to use the sequences of four coin flips that contain 1 and 2 heads to find the sequences of five coin flips that contain 2 heads.
 To the sequences of coin flips that have one head, add an H at the end. To the sequences of coin flips that have two heads, add a T at the end. Students should be encouraged to make the connection to the Pascal's Triangle recursion.

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9. You are currently at the intersection of 1st Ave. and 1st St., heading northeast to the math-themed restaurant, *The Improper Fraction* at 4th Ave. and 3rd St. Find a correspondence between walking paths and sequences of coin flips.



- (a) Which sequence of coin flips corresponds to the walking path above?

HTTHT or THHTH

- (b) Which subset corresponds to the walking path above?

A, D or B, C, E