Binomial Probabilities

University of Minnesota

Preliminaries and Objectives

Preliminaries

- Basic Probability (AND, OR, NOT)
- Binomial Theorem expanding $(x + y)^n$
- Pascal's Triangle
- Combinations
- Random Variables

Objectives

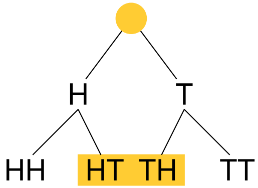
- Calculate probabilities in successive trials with only two outcomes, either succeed or fail.

Flip one coin

$$P(\text{heads}) = \frac{1}{2} \qquad P(\text{tails}) = \frac{1}{2}$$

Flip two coins

		Second Flip	
		Heads	Tails
First Flip	Heads	HH	HT
	Tails	TH	TT



$$P(\text{two heads}) = \frac{1}{4}$$

$$P(\text{one head, one tail}) = \frac{2}{4}$$

$$P(\text{zero heads, two tails}) = \frac{1}{4}$$

Random Variable

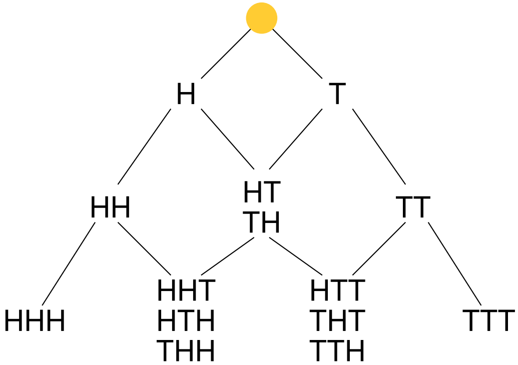
Let $H =$ the number of heads on two flips of a coin

$$P(H = 2) = \frac{1}{4}$$

$$P(H = 1) = \frac{2}{4}$$

$$P(H = 0) = \frac{1}{4}$$

Flip three coins



$$P(H=3)=\frac{1}{8},\;\; P(H=2)=\frac{3}{8},\;\; P(H=1)=\frac{3}{8},\;\; P(H=0)=\frac{1}{8}$$

Pascal's Triangle - Flip five coins

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

$$P(H = 5) = \frac{1}{32} \qquad P(H = 4) = \frac{5}{32} \qquad P(H = 3) = \frac{10}{32}$$

$$P(H = 2) = \frac{10}{32} \qquad P(H = 1) = \frac{5}{32} \qquad P(H = 0) = \frac{1}{32}$$

General Formula for Coin Flips

Flip n coins, what is the probability that exactly k land heads?

The numerator is the k^{th} number in row n of Pascal's Triangle.

1 5 10 10 5 1

The denominator is 2^n

$$P(H = k) = \frac{{}_nC_k}{2^n} = \frac{C(n,k)}{2^n} = \frac{\binom{n}{k}}{2^n}$$

Example 1

Flip 7 coins, what is the probability that exactly 5 land heads?

1	7	21	35	35	21	7	1

$$P(H = 5) = \frac{21}{128} \approx 0.164$$