

Unions, Intersections, and Complements in Probability



Preliminaries and Objectives

Preliminaries

- Techniques of Counting
- Definition of Probability

Objectives

- Find probabilities of events combined using AND, OR, NOT

Unions

Example: Roll two dice. What is the probability that the total of the two dice is either 7 or 11?

						
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12

Example: Roll two dice. What is the probability that the total of the two dice is either 7 or 11?

$E = \text{total is 7}$ $P(E) = \frac{6}{36}$ E and F are mutually exclusive

$F = \text{total is 11}$ $P(F) = \frac{2}{36}$

$$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36}$$

Conditional Probability

Sometimes the occurrence of an event changes our mind about the probability of another event.

$E =$ roll a 

$F =$ roll ≥ 10 on two dice

$$P(E) = \frac{1}{6}$$

$$P(F) = \frac{6}{36}$$



Conditional Probability

Sometimes the occurrence of an event changes our mind about the probability of another event.

E = roll a 

F = roll ≥ 10 on two dice

$$P(E) = \frac{1}{6}$$

$$P(F) = \frac{6}{36}$$















$$P(F | E) = \frac{3}{6}$$

Conditional Probability

						
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12


Independence

If $P(F | E) = P(F)$, then E and F are independent.


Intersections


If the occurrence of event E has no effect on the occurrence of event F , then E and F are said to be **independent**.

When rolling two dice, what is the probability that both are  ?

						
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
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	7	8	9	10	11	12

Intersections

When rolling two dice, what is the probability that both are  ?

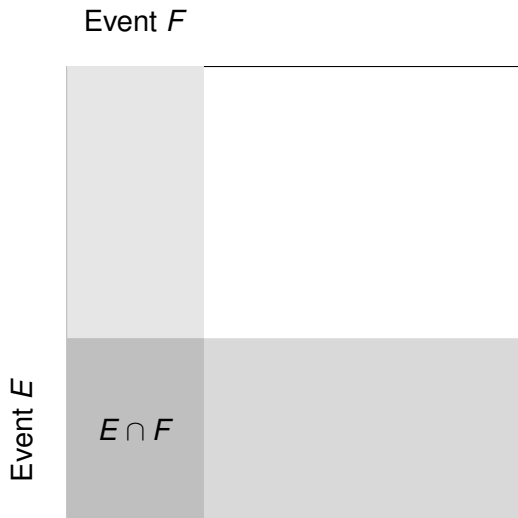
$$E = \text{$$

$$F = \text{$$

$$P(E \text{ and } F) = P(E \cap F) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$P(E \text{ and } F) = P(E \cap F) = P(E) \cdot P(F)$$

Intersections




Complements

There is a 40% chance that it will rain today. What is the chance that it will not rain today?

Answer: $P(\text{no rain}) = 1 - P(\text{rain}) = 60\%$

Unions of Independent Events

When rolling two dice, what is the probability that at least one of the dice is ?

						
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12

$$P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$$

Examples

When picking a card from a standard deck, what is the probability that ...

- 1 the card is either a ♠ or ♣?
- 2 the card is 7 and ♥?
- 3 the card is not a King?
- 4 the card is either a 7 or a ♠?

K♠	K♥	K♦	K♣
Q♠	Q♥	Q♦	Q♣
J♠	J♥	J♦	J♣
10♠	10♥	10♦	10♣
9♠	9♥	9♦	9♣
8♠	8♥	8♦	8♣
7♠	7♥	7♦	7♣
6♠	6♥	6♦	6♣
5♠	5♥	5♦	5♣
4♠	4♥	4♦	4♣
3♠	3♥	3♦	3♣
2♠	2♥	2♦	2♣
A♠	A♥	A♦	A♣

Examples

When picking a card from a standard deck, what is the probability that ...

- 1 the card is either a ♠ or ♣?
- 2 the card is 7 and ♥?
- 3 the card is not a King?
- 4 the card is either a 7 or a ♠?

Answers:

$$1 \quad \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$

$$2 \quad \frac{1}{13} \cdot \frac{1}{4} = \frac{1}{52}$$

$$3 \quad 1 - \frac{1}{13} = \frac{12}{13}$$

$$4 \quad \frac{1}{13} + \frac{1}{4} - \frac{1}{13} \cdot \frac{1}{4} = \frac{4}{52} + \frac{13}{52} - \frac{1}{13} \cdot \frac{1}{4} = \frac{16}{52}$$

Recap

- Intersections - “AND” - multiply
 $P(E \cap F) = P(E) \cdot P(F)$ when independent
- Unions - “OR” - add
 $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
- Complements - “NOT” - subtract from 1
 $P(\text{not } E) = 1 - P(E)$