Binomial Probabilities

Preliminaries and Objectives

Preliminaries
- Basic Probability (AND, OR, NOT)
- Binomial Theorem expanding \((x + y)^n\)
- Pascal's Triangle
- Combinations
- Random Variables

Objectives
- Calculate probabilities in successive trials with only two outcomes, either succeed or fail.

Flip one coin

\[ P(\text{heads}) = \frac{1}{2} \quad P(\text{tails}) = \frac{1}{2} \]

Flip two coins

<table>
<thead>
<tr>
<th>First Flip</th>
<th>Second Flip</th>
<th>Heads</th>
<th>Tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heads</td>
<td>HH</td>
<td>HT</td>
<td>TH</td>
</tr>
<tr>
<td></td>
<td>HH</td>
<td>HT</td>
<td>TT</td>
</tr>
</tbody>
</table>

\[
P(\text{two heads}) = \frac{1}{4} \\
P(\text{one head, one tail}) = \frac{2}{4} \\
P(\text{zero heads, two tails}) = \frac{1}{4}
\]

Flip three coins

\[
P(H = 3) = \frac{1}{8} \\
P(H = 2) = \frac{3}{8} \\
P(H = 1) = \frac{3}{8} \\
P(H = 0) = \frac{1}{8}
\]

Random Variable

Let \(H\) be the number of heads on two flips of a coin

\[
P(H = 2) = \frac{1}{4} \\
P(H = 1) = \frac{2}{4} \\
P(H = 0) = \frac{1}{4}
\]

Pascal's Triangle - Flip five coins

General Formula for Coin Flips

Flip \(n\) coins, what is the probability that exactly \(k\) land heads?

The numerator is the \(k\)th number in row \(n\) of Pascal's Triangle.

\[
\begin{array}{c}
10 \\
15 \\
20 \\
25 \\
30 \\
35 \\
40 \\
45 \\
50 \\
55 \\
60 \\
65 \\
70 \\
75 \\
80 \\
85 \\
90 \\
95 \\
100
\end{array}
\]

\[
P(H = 5) = \frac{1}{32} \\
P(H = 4) = \frac{5}{32} \\
P(H = 3) = \frac{10}{32} \\
P(H = 2) = \frac{10}{32} \\
P(H = 1) = \frac{5}{32} \\
P(H = 0) = \frac{1}{32}
\]

Example 1

Flip 7 coins, what is the probability that exactly 5 land heads?

\[
\begin{array}{c}
1 \\
7 \\
21 \\
35 \\
35 \\
21 \\
7 \\
1
\end{array}
\]

\[
P(H = 5) = \frac{21}{128} \approx 0.164
\]